17. Some Prehomogeneous Vector Spaces with Relative Invariants of Degree Four and the Formula of the Fourier Transforms

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In this article, we shall investigate the relative invariant $f(x)$ of a regular prehomogeneous vector space (G, V) when it is one of the following ones; 1) $SL(6)\times GL(1)$ $(\Lambda_3\times\Lambda_1)$, 2) $Sp(3)\times GL(1)$ $(\Lambda_3\times\Lambda_1)$, 3) $Spin(12)\times GL(1)$ ((half-spin rep.)$\times\Lambda_1$), 4) $E_7\times GL(1)$ ((56 dim. rep.)$\times\Lambda_1$), where Λ_i is the representation on the space of the skew-symmetric tensors of rank i. The polynomial $f(x)$ has the following form,

(1) $$f(x)=(x_0y_0-\langle X, Y\rangle)^2+4x_0N(Y)+4y_0N(X)-4\langle X^{\#}, Y^{\#}\rangle.$$

Here, $x=(x_0, y_0, X, Y)\in C\oplus C\oplus C^m\oplus C^m$ and $\langle X, Y\rangle$ is some bilinear form in X and Y, $N(X)$ is some polynomials in X, and $X\mapsto X^{\#}$ is some polynomial mapping from the X-space into itself.

We shall calculate the Fourier transform of the hyperfunction $|f(x)|^s$ for a generic $s\in C$. As shown in [5], the formula of the Fourier transform gives the functional equation of the local zeta function associated with the prehomogeneous vector spaces.

1. Let $u_1, \cdots, u_6$ be a basis of the six-dimensional complex vector space E with the natural action of $G=SL(6)\times GL(1)$, i.e., $(u_1, \cdots, u_6)\mapsto C_2(u_1, \cdots, u_6)^tg_1$ for $(g_1, c)\in SL(6)\times GL(1)$. We denote by $V(20)$ the vector space of the skew-symmetric tensors on E of rank 3 and x_{ijk} denotes the coefficient of $u_i\wedge u_j\wedge u_k$. The complex algebraic group $SL(6)\times GL(1)$ acts on $V(20)$, and it is a regular prehomogeneous vector space. We identify $V(20)$ and $C\oplus C\oplus M(3, C)\oplus M(3, C)$ by

(2) $$x_0=x_{123} \qquad y_0=x_{456}$$
$$X=\begin{pmatrix} x_{423}, & x_{143}, & x_{124} \\ x_{523}, & x_{153}, & x_{125} \\ x_{623}, & x_{163}, & x_{126}\end{pmatrix} \qquad Y=\begin{pmatrix} x_{156}, & x_{416}, & x_{451} \\ x_{256}, & x_{426}, & x_{452} \\ x_{356}, & x_{436}, & x_{453}\end{pmatrix}.$$

By setting $\langle X, Y\rangle=\mathrm{tr}\,(X\cdot Y)$, $N(X)=\det X$, and $X^{\#}=$ the cofactor matrix of X, $f(x)$ is an irreducible relatively invariant polynomial on the prehomogeneous vector space $(G, V)=(SL(6)\times GL(1), V(20))$ with the character $\chi(g_1, c)=c^{12}$. This is the prehomogeneous vector space 1). We define the symplectic group $Sp(3)$ as the subgroup of $SL(6)$ consisting of the elements which leave $u_1\wedge u_4+u_2\wedge u_5+u_3\wedge u_6$ invariant. When we set

(3) $$V(14)=\{(x_0, y_0, X, Y)\in V(20);\ {}^tX=X,\ {}^tY=Y\},$$