16. *Cauchy Problem for Hyperbolic Differential Operators with Double Characteristic Roots*

By Katsuju IGARI
Department of Applied Mathematics, Ehime University

(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 12, 1980)

1. Introduction. Let us consider the Cauchy problem for hyperbolic differential operators with double characteristic roots. Already we have some sufficient conditions for this Cauchy problem to be well-posed in C^∞-class, cf. [5]. On the other hand, we also know that without such a condition, this Cauchy problem is well-posed in κ-Gevrey class, $1\le\kappa<2$, cf. [2], [3].

In this paper, we introduce a number $\kappa^*\in[2,\infty]$ which shall be determined according to a given operator and show that if $1\le\kappa<\kappa^*$, the above Cauchy problem is well-posed in κ-Gevrey class.

2. Definitions. We consider the Cauchy problem

$$\text{(C)}\quad \begin{cases} P[u]=D_t^m u+\sum_{j=0}^{m-1}\sum_{|\nu|\le m-j} a_{\nu j}D_x^\nu D_t^j u=f(x,t), & (x,t)\in\Omega \\ D_t^j u|_{t=0}=\phi_j(x), \quad j=0,1,\cdots,m-1 \end{cases}$$

where $\Omega=\boldsymbol{R}^n\times[0,T]$, $T>0$, $D_t=-i\frac{\partial}{\partial t}$, $D_j=-i\frac{\partial}{\partial x_j}$, $\nu=(\nu_1,\cdots,\nu_n)$; ν_j are non-negative integers, $D_x^\nu=D_1^{\nu_1}\cdots D_n^{\nu_n}$. Let $P_j(x,t;D_x,D_t)$ be the homogeneous part of degree j in (D_x,D_t) of $P(x,t;D_x,D_t)$.

We say that $a(x,t)$ belongs to a class $\gamma^{(\kappa)}$, $\phi(x)$ to $\Gamma^{(\kappa)}$ and $\psi(x,t)$ to $\Gamma^{r(\kappa)}$; $r=0,1,\cdots,\infty$, if there exist constants $\rho>0$ and $C\ge 0$ according to $a(x,t)$, $\phi(x)$ and $\psi(x,t)$ respectively such that

$$|D_x^\nu D_t^j a(x,t)|\le C\frac{(j+|\nu|)!^\kappa}{\rho^{j+|\nu|}},\ (x,t)\in\Omega,\qquad \text{for any } j \text{ and } \nu,$$

$$\|D_x^\nu\phi\|\le C\frac{|\nu|!^\kappa}{\rho^{|\nu|}},\qquad \text{for any } \nu,$$

$$\|D_x^\nu D_t^j\psi(t)\|\le C\frac{(j+|\nu|)!^\kappa}{\rho^{j+|\nu|}},\ 0\le t\le T,\qquad \text{for any } j\le r \text{ and any } \nu,$$

respectively, where $\|\ \|$ denotes the L_x^2-norm.

We also say that $h(x,t,\xi)$ belongs to $\mathcal{B}_t^k[S^r(\kappa)]$ if 1) $h(x,t,\xi)$ is homogeneous of degree r in ξ, and 2) there exists a constant $\rho>0$ such that for any $j\le k$ and any α, β,

$$|D_t^j D_x^\beta D_\xi^\alpha h(x,t,\xi)|\le C_{j\alpha}\frac{|\beta|!^\kappa}{\rho^{|\beta|}},\qquad (x,t)\in\Omega,\ |\xi|=1,$$

where $C_{j\alpha}$ is a constant independent of β.

3. Result. We assume the following three conditions: