13. *A Remark on the Inverse Theorem of Cauchy-Kowalevski**)

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1. Introduction. The purpose of this paper is to show the inverse theorem of Cauchy-Kowalevski. Consider the following Cauchy problem in a neighbourhood of the origin of C^{n+1}:

(1.1) $$a(D_x, D_t)u(x, t) = f(x, t),$$

(1.2) $$D_t^k u|_{t=0} = u_k(x), \qquad 0 \leqq k < m,$$

where $(x, t) = (x_1, \cdots, x_n, t) \in C^{n+1}$ and $(D_x, D_t) = (\partial/\partial x_1, \cdots, \partial/\partial x_n, \partial/\partial t)$.

As well-known, Cauchy-Kowalevski's theorem says that if the operator $a(D_x, D_t)$ is Kowalevskian of order m with respect to D_t, that is,

(1.3) $$a(D_x, D_t) = D_t^m + \sum_{j=1}^{m} a_j(D_x) D_t^{m-j}, \qquad \text{order } a_j(D_x) \leqq j,$$

then there exists a unique holomorphic solution $u(x, t)$ at the origin for any holomorphic function $f(x, t)$ at the origin and any holomorphic Cauchy data $\{u_k\}_{k=0}^{m-1}$ at the origin.

Our purpose is to show the converse. That is,

Theorem 1. *The Cauchy-Kowalevski theorem holds for the problem* (1.1)–(1.2) *if and only if* $a(D_x, D_t)$ *is Kowalevskian of order* m *with respect to* D_t.

Concerning this problem, some results were obtained in the case when $a(x, t; D_x, D_t) = D_t^m + \sum_{j=1}^m a_j(x, t; D_x) D_t^{m-j}$ (see Mizohata [6], [7], Miyake [2] and Kitagawa-Sadamatsu [1]). In the case of system of partial differential equations which is written in a normal form with respect to the time variable, the author [3] and Mizohata [5] obtained some results. Our interest here is to consider the problem without such restriction on the operator $a(D_x, D_t)$.

Our theorem will be proved in § 3, by means of Theorem 2 which concerns the Goursat problem. We have to mention that our theorem can not be extended to the case of variable coefficients by the same method as in this paper.

2. Holomorphic Goursat problem. Consider the following Goursat problem in $C_x^n \times C_t^1$:

(2.1) $$D_x^\alpha D_t^m u(x, t) = \sum_{j=0}^{m} a_j(D_x) D_t^{m-j} u(x, t) + f(x, t),$$

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