12. *Quasi-Periodic Solutions of Nonlinear Equations of sine-Gordon Type and Fixed Point Free Involutions of Hyperelliptic Curves*

By Etsuro DATE
Department of Mathematics, Osaka University

(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 12, 1980)

1. The connection between the theory of nonlinear equations solvable by the inverse scattering method and the theory of Riemann surfaces (algebraic curves) has been discussed by many authors (see, for example, [6], [3], [8], [9]).

In this note we consider quasi-periodic solutions of the sine-Gordon equation

$$u_{\xi\eta}+\sin u=0 \tag{1}$$

and the equation of the system of Pohlmeyer [10] and Lund-Regge [7]

$$\begin{aligned} &u_{\xi\eta}-v_\xi v_\eta \sin(u/2)/2\cos^3(u/2)+\sin u=0,\\ &v_{\xi\eta}+(u_\xi v_\eta+u_\eta v_\xi)/\sin u=0 \end{aligned} \tag{2}$$

which is a generalization of (1); if $v=$constant, then (2) reduces to (1). We show that the solutions of (2) which correspond to the hyperelliptic curves admitting fixed point free involutions reduce to the solutions of (1).

Quasi-periodic solutions of (1) were discussed by Kozel-Kotlyarov [5] and by Its [4]. With the aid of the representation of (1) as the compatibility condition of the linear differential equations

$$\begin{aligned} &i\Psi_\xi+2^{-1}u_\xi\begin{pmatrix}1&0\\0&-1\end{pmatrix}\Psi+2^{-1}\zeta\begin{pmatrix}0&1\\1&0\end{pmatrix}\Psi=0,\\ &i\Psi_\eta+2^{-1}\zeta^{-1}\begin{pmatrix}0&\exp(iu)\\\exp(-iu)&0\end{pmatrix}\Psi=0, \qquad \zeta\in \boldsymbol{C}, \end{aligned} \tag{3}$$

they showed that quasi-periodic solutions of (1) correspond to the hyperelliptic curves

$$w^2=z\prod_{j=1}^{2g}(z-z_j). \tag{4}$$

They also showed that the simultaneous solution Ψ of (3) and the parameter ζ in (3) are two-valued functions on the Riemann surface of (4).

First we construct quasi-periodic solutions of (2) by a method similar to that of Kricheber [6], starting from the curves $\mu^2=\prod_{j=1}^{2g+2}(\lambda-\lambda_j)$. Next we show that the solutions of (2) constructed in this way reduce to the solutions of (1) when we specialize the curves to the form

$$\mu^2=\prod_{j=1}^{2g}(\lambda-\lambda_j)(\lambda+\lambda_j). \tag{5}$$

This assertion is proved by using a fixed point free involution (λ, μ)