102. *On the Regularity of Arithmetic Multiplicative Functions. I*

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1. Statement of result. An arithmetical function $f(n)$ is called *additive* (resp. *multiplicative*), if $f(mn)=f(m)+f(n)$ (resp. $f(mn)=f(m)f(n)$) for any pair m, n of relatively prime natural numbers, and is called *completely additive* (resp. *completely multiplicative*), if the above equality holds for every pair m, n.

We have several sufficient conditions under which an additive arithmetical function turns out to be completely additive. P. Erdös ([1]) proved that an additive arithmetical function which satisfies the condition

$$\lim_{n\to\infty}\{f(n+1)-f(n)\}=0, \tag{1.1}$$

is completely additive (and, more than that, it is equal to $c(\log n)$ for some c).

I. Kátai ([2]) succeeded in replacing (1.1) by a weaker condition,

$$\lim_{x\to\infty}\frac{1}{x}\sum_{n\leqslant x}|f(n+1)-f(n)|=0. \tag{1.2}$$

F. Skof ([3]) gave another condition: an additive arithmetical function which satisfies

$$\lim_{\substack{n\to\infty\\ n\notin S}}\{f(n+1)-f(n)\}=0, \tag{1.3}$$

where S is a sequence of density zero, is also completely additive.

We shall prove here a theorem which will show clearly the link between F. Skof's and I. Kátai's results.

Theorem. *Suppose $g(n)$ is a multiplicative arithmetical function such that:*

$$\lim_{x\to\infty}\frac{1}{x}\sum_{\substack{n\leqslant x\\ n\notin S}}|g(n+1)-g(n)|=0, \quad \textit{and} \quad |g(n)|=1, \tag{1.4}$$

where S is a sequence of density zero. Then $g(n)$ is completely multiplicative.

2. Proof of the theorem. Lemma. *Let $\{t_n\}_{n=1}^{\infty}$ be an increasing sequence of integers. Suppose $\limsup_{n\to\infty} t_n/n<\infty$, then, under the assumption of our theorem, we get*

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