93. On Certain Numerical Invariants of Mappings over Finite Fields. II

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Introduction. This is a continuation of the first paper [1] which will be referred to as (I) in this paper.*) Our purpose here is to determine invariants ρ_F, σ_F (see (I.1.1), (I.1.6)) for quadratic mappings $F: X \to Y$ of vector spaces over a finite field $K=F_q$ (q : odd) with respect to the quadratic character of the multiplicative group of K. In particular, we shall obtain explicit values of invariants for such mappings arising from pairs of quadratic forms.

§1. Quadratic mappings. Let K be the finite field with q elements: $K=F_q$ (q : odd). Denote by χ the character of $K^\times$ of order 2. As usual, we extend χ to K by $\chi(0)=0$. Let X, Y be vector spaces over K of dimension n, m, respectively, and $F: X \to Y$ be a quadratic mapping. By definition, $F_\lambda = \lambda \circ F$ is a quadratic form on X for every linear form $\lambda \in Y^*$. By (I.1.6), we have

(1.1) $\sigma_F = \sum_{\lambda \in Y^*} |S_{F_\lambda}|^2$,

where

(1.2) $S_{F_\lambda} = \sum_{x \in X} \chi(F_\lambda(x))$.

Thanks to the following lemma, proof of which is left to the reader as an exercise, the determination of σ_F is much easier than that of ρ_F.

(1.3) **Lemma.** *Let V be a vector space of dimension r over K and Q be a non-degenerate quadratic form on V. Then we have*

$$S_Q = \sum_{x \in V} \chi(Q(x)) = \begin{cases} 0, & \text{if } r \text{ is even}, \\ (q-1)q^{(r-1)/2}\chi((-1)^{(r-1)/2}\det Q), & \text{if } r \text{ is odd}. \end{cases}$$

(1.4) **Theorem.** *Let $K=F_q$ (q : odd). Let F be a quadratic mapping $X \to Y$ of vector spaces over K, $n=\dim X$, $m=\dim Y$. Let r_λ be the rank of the quadratic form $F_\lambda = \lambda \circ F$, $\lambda \in Y^*$. Then, we have*

$$\rho_F = q^{n-m}(q-1) \sum_{r_\lambda \text{ odd}} q^{n-r_\lambda}.$$

Proof. Write F_λ as a diagonal form $a_1x_1^2 + \cdots + a_{r_\lambda}x_{r_\lambda}^2$, $a_i \in K^\times$. By (1.3), we have

$$S_{F_\lambda} = \sum_{x \in X} \chi(a_1x_1^2 + \cdots + a_{r_\lambda}x_{r_\lambda}^2)$$
$$= \sum_{(x_{r_\lambda+1}, \cdots, x_n)} \sum_{(x_1, \cdots, x_{r_\lambda})} \chi(a_1x_1^2 + \cdots + a_{r_\lambda}x_{r_\lambda}^2)$$

*) For example, we mean by (I.2.3) the item (2.3) in (I).