# 92. *On Semipositive Line Bundles*

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Unlike the positivity of line bundles, in which case several different definitions turn to be equivalent to each other, there are a couple of really different notions of semipositivity. Here we want to clarify the situation as well as possible now. Details and proofs will be published elsewhere.

For the sake of simplicity we work in the category of projective $K$-schemes, where $K$ is an algebraically closed field. $K$ is assumed to be the complex number field in the statements indicated by $/C$. A variety means an irreducible, reduced projective $K$-scheme. Tensor products of line bundles are denoted additively, and are regarded as rational equivalence classes in Chow ring.

**§1. Definitions and interrelations.**

(1.1) **Definition.** Let $S$ be a projective $K$-scheme and let $L$ be a line bundle on $S$. Then $L$ is said to be

a) *semiample*, if $\mathcal{O}_S[mL]$ is generated by global sections for some $m>0$;

b) *cohomologically semipositive* (abbreviation: *c-semipositive*), if, for any coherent sheaf $\mathcal{F}$ on $S$ and for any very ample line bundle $H$ on $S$, there is an integer $a$ such that $H^p(\mathcal{F}[tL+sH])=0$ for any $p>0$, $t\geqq 0$, $s>a$;

c) *approximately ample*, if there is a line bundle $F$ on $S$ and a positive integer $m$ such that $\mathcal{O}_S[F+tmL]$ is generated by global sections for any $t\geqq 0$;

d) *numerically semipositive* (abbr: *n-semipositive*), if $LC\geqq 0$ for any curve $C$ in $S$;

e) *universally effective*, if, for any subvariety $V$ of $S$, there exists a positive integer $m$ such that $H^0(V', mL_{V'})\neq 0$, where $V'$ is the normalization of $V$;

f) *geometrically semipositive* (abbr: *g-semipositive*), if $S$ is a complex manifold and $c_1(L)$ is represented by a closed Hermitian (1, 1)-form which is everywhere positive semidefinite.

(1.2) **Theorem.** *All the above notions* a)–f) *satisfy the following axioms.*

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