91. *Free Arrangements of Hyperplanes and Unitary Reflection Groups*[*)]

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1. Free arrangements. We call a non-void finite family of hyperplanes in C^{n+1} (or $P^{n+1}(C)$) an *affine* (resp. *projective*) *n-arrangement.* A set X is simply called an *n-arrangement* if X is either an affine n-arrangement or a projective n-arrangement. An n-arrangement X is called to be *central* when $\bigcap_{H\in X} H \neq \phi$. Denote $\bigcup_{H\in X} H$ by $|X|$.

Let X be a central affine n-arrangement. By an appropriate translation of the origin we can assume that $\bigcap_{H\in X} H$ contains the origin O in C^{n+1}. Let $Q\in C[z_0, \cdots, z_n]$ be a square-free defining equation of $|X|$. By $\mathcal{O}$ denote we $\mathcal{O}_{C^{n+1},O}$. Then

$D(X):=\{\theta$; a germ at the origin of holomorphic vector fields such that $\theta\cdot Q\in Q\cdot\mathcal{O}\}$

is an $\mathcal{O}$-module. We call X to be *free* if $D(X)$ is a free $\mathcal{O}$-module.

Assume that a central affine n-arrangement X is free. Let $\{\theta_0, \cdots, \theta_n\}$ be a system of free basis for $D(X)$ such that each θ_i is homogeneous of degree d_i. (θ_i is homogeneous of degree d_i if θ_i has an expression

$$\theta_i=\sum_{j=0}^{n} f_j(\partial/\partial z_j),$$

where each $f_j\in C[z_0, \cdots, z_n]$ is either 0 or homogeneous of degree d_i.) We call the integers $(d_0, \cdots, d_n)$ the *generalized exponents* of X. They depend only on X [7].

Let X be a projective n-arrangement. Denote $P^{n+1}(C)$ simply by P^{n+1}. Let $Q\in C[z_0, \cdots, z_{n+1}]$ be a homogeneous polynomial defining a set $|X|\subset P^{n+1}$. Then there exists a unique central affine $(n+1)$-arrangement $\tilde{X}$ such that

$$V(Q)=|\tilde{X}|\subset C^{n+2}.$$

We call X to be *free* if $\tilde{X}$ is free.

Assume that a projective n-arrangement X is free. Let $(d_0, d_1, \cdots, d_n)$ be the generalized exponents of $\tilde{X}$, then we can assume that $d_0=1$ (due to the existence of the Euler vector field

$$\sum_{i=0}^{n} z_i(\partial/\partial z_i)).$$

The *generalized exponents* of X are defined to be $(d_1, \cdots, d_n)$.

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