89. *On the Essential Boundary and Supports of Harmonic Measures for the Heat Equation*

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1. For the heat equation on an arbitrary bounded domain D located in the $(n+1)$-dimensional Euclidean space $R^{n+1}(=R^n\times R)$ $(n\geqq 1)$, we can solve the Dirichlet problem in the sense of Perron-Wiener-Brelot. Because of this, there exists the harmonic measure ω_p on the boundary ∂D for every $p\in D$. The support $\mathrm{supp}\,(\omega_p)$ of ω_p, however, does not coincide with ∂D, so the Dirichlet problem and the minimum principle should be considered on a relevant part of the boundary. From the standpoint of the Dirichlet problem, an intrinsic part of the boundary would be $\overline{\bigcup_{p\in D}\mathrm{supp}\,(\omega_p)}$. This is also available for the minimum principle of superharmonic functions (see Corollary 9).

On the other hand, for the heat equation two kinds of significant boundary part are known. One is the parabolic boundary $\partial_p D$ and the other is the essential boundary $\mathrm{ess}\,(\partial D)$ (see Definitions 2 and 3).

Our purpose of this paper is to show that our relevant parts $\overline{\bigcup_{p\in D}\mathrm{supp}\,(\omega_p)}$, $\mathrm{ess}\,(\partial D)$ and $\overline{\partial_p D}$ are equal except for negligible sets. This throws light on a geometrical property of $\overline{\bigcup_{p\in D}\mathrm{supp}\,(\omega_p)}$.

Theorem 1. (1) $\mathrm{ess}\,(\partial D)=\overline{\partial_p D}$.

(2) $\mathrm{ess}\,(\partial D)\supset\overline{\bigcup_{p\in D}\mathrm{supp}\,(\omega_p)}$ *and* $Z=\mathrm{ess}\,(\partial D)\setminus\overline{\bigcup_{p\in D}\mathrm{supp}\,(\omega_p)}$ *is polar.*[1] *Furthermore* $Z\subset D^*\setminus D$, *where* D^* *is the interior of the closure of* D.[2]

2. Since a bounded domain D associated with the heat equation is a Bauer harmonic space, we follow C. Constantinescu and A. Cornea [1] for basic notation and terminology in potential theory.

We denote by (x,t) a point p in R^{n+1}, where $x=(x_1,\cdots,x_n)$ are the space variables and t the time variable.

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1) This means that there exists a positive measure μ on R^{n+1} such that the potential $G\mu(p)=\int G(p,q)d\mu(q)$ on R^{n+1} takes the value $+\infty$ on Z, where for $p=(x,t)$, $q=(y,s)$

$$G(p,q)=\begin{cases}(4\pi(t-s))^{-n/2}\exp(-\|x-y\|/4(t-s)) & \text{if } t>s\\ 0 & \text{if } t\leqq s,\end{cases}$$

and $\|x\|$ denotes the Euclidean norm of $x\in R^n$.

2) This is an affirmative solution of a question proposed orally by Professor M. Itô.