85. *Explicit Formulae for Solutions of Schrödinger Equations with Quadratic Hamiltonians*

By Kimimasa NISHIWADA
Research Institute for Mathematical Sciences,
Kyoto University

(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 13, 1980)

This note is concerned with a global, explicit construction of the fundamental solution for the Cauchy problem

$$(1)\qquad \begin{aligned} &\frac{\partial}{i\partial t}\psi(t,x)=H\Big(t,x,\frac{\partial}{i\partial x}\Big)\psi(t,x), \qquad (t,x)\in \boldsymbol{R}^{n+1},\\ &\psi(0,x)=\psi(x), \end{aligned}$$

with a quadratic Hamiltonian H:

$$\begin{aligned} H(t,x,\xi)=&\frac{1}{2}\langle\alpha(t)\xi,\xi\rangle+\langle\beta(t)x,\xi\rangle+\frac{1}{2}\langle\gamma(t)x,x\rangle\\ &+\langle a_1(t),\xi\rangle-\langle a_2(t),x\rangle+c(t), \end{aligned}$$

where α,β,γ are real $n\times n$ matrices with α,γ symmetric and $a_1, a_2\in\boldsymbol{R}^n$, $c\in\boldsymbol{R}$. All the coefficients are assumed to be continuously dependent on t.

We shall construct a unitary map $\mathcal{U}_t:\psi(x)\to\psi(t,x)$ in $L^2(\boldsymbol{R}^n)$ which is strongly continuous in t and has an explicit form written by means of one or two integral transformations. One can obtain solutions for a more general class of Hamiltonians at the expense of such explicitness (Fujiwara [1], Kitada and Kumano-go [3]). As our tools we shall make use of Maslov indices and representations of the Heisenberg group and the metapletic group. Although they are well established facts, it would bear some meaning to review them in this connection.

To make $H(t,x,D)$ a symmetric operator, we shall assign to $\langle\beta x,\xi\rangle$ the operator

$$\frac{1}{2}(\langle\beta x,D\rangle+\langle D,\beta x\rangle),$$

while to the rest of the symbol an operator in the usual way. One can write $H=H_1+H_2$ with

$$H_1(t,x,\xi)=-\sigma(a,(x,\xi))+c(t)$$

where $a=(a_1,a_2)$ and σ denotes the symplectic form $\sigma((x,\xi),(x',\xi'))=\langle x',\xi\rangle-\langle x,\xi'\rangle$.

1. Heisenberg group. When a is independent of t, the fundamental solution of

$$(2)\qquad D_t\psi=H_1(t,x,D)\psi,\qquad \psi(0,x)=\psi(x)\in\mathcal{S}(\boldsymbol{R}^n)$$

is given by $V_t(a,c)=e^{i\tilde{c}(t)}V(ta)$, where