71. *Singular Hadamard's Variation of Domains and Eigenvalues of the Laplacian*

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§1. **Introduction.** Let Ω be a bounded domain in $\mathbf{R}^n$ with C^3 boundary γ and w be a fixed point in Ω. For any sufficiently small $\varepsilon>0$, let B_ε be the ball defined by

$$B_\varepsilon=\{z\in\Omega\,;\,|z-w|<\varepsilon\}.$$

Let Ω_ε be the bounded domain defined by $\Omega_\varepsilon=\Omega\backslash\bar{B}_\varepsilon$. Then the boundary of Ω_ε consists of γ and ∂B_ε.

Let $0>\mu_1(\varepsilon)\geq\mu_2(\varepsilon)\geq\cdots$ be the eigenvalues of the Laplacian with the Dirichlet condition on $\gamma\cup\partial B_\varepsilon$. And let $0>\mu_1\geq\mu_2\geq\cdots$ be the eigenvalues of the Laplacian in Ω with the Dirichlet condition on γ. We arrange them repeatedly according to their multiplicities.

The main aim of this note is to give an asymptotic expression of $\mu_j(\varepsilon)$ when ε tends to zero.

We have the following

Theorem 1. *Let Ω be a bounded domain in $\mathbf{R}^2$ with C^3 boundary γ. Fix j. Assume that the multiplicity of μ_j is equal to one, then*

$$\mu_j(\varepsilon)-\mu_j=-2\pi(\log(1/\varepsilon))^{-1}\varphi_j(w)^2+O((\log(1/\varepsilon))^{-2}) \tag{1.1}$$

holds when ε tends to zero. Here φ_j denotes the eigenfunction of the Laplacian with the Dirichlet condition on γ satisfying

$$\int_\Omega\varphi_j(x)^2dx=1.$$

For the case $n=3$, we have the following

Theorem 2. *Let Ω be a bounded domain in $\mathbf{R}^3$ with C^3 boundary γ. Fix j. Assume that the multiplicity of μ_j is equal to one, then*

$$\mu_j(\varepsilon)-\mu_j=-4\pi\varepsilon\varphi_j(w)^2+O(\varepsilon^{3/2}) \tag{1.2}$$

holds when ε tends to zero. Here φ_j denotes the normalized eigenfunction associated with μ_j.

In §2 we give a rough sketch of proof of Theorem 1. To prove Theorem 1 we employ the singular Hadamard variational formula for the Green's function of the Laplacian due to [5]. The details of this paper will be given in [4].

§2. **Outline of proof of Theorem 1.** In this section we give a rough sketch of proof of Theorem 1.

Let $G(x,y)$ be the Green's function on Ω, that is, it satisfies the following: