70. *Deformation of Linear Ordinary Differential Equations. IV*

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In this note exploiting quantum field operators we construct an isomonodromy family with a prescribed monodromy data. This approach was initiated by Sato, Miwa and Jimbo [1] in the case of regular singularities. As for irregular singularities some special cases have been treated in [2], [3]. Here we consider the following general case; we construct an $m\times m$ matrix $Y(x_0, x)$ normalized as $Y(x_0, x_0)=1$ which enjoys the monodromy property with respect to x with the following monodromy data [4], [5]

$$(1)\qquad \begin{aligned} &a_1;\, T^{(1)}_{-r_1}, \cdots, T^{(1)}_0, S^{(1)}_1, \cdots, S^{(1)}_{2r_\nu}, C^{(1)},\\ &\qquad\vdots\\ &a_n;\, T^{(n)}_{-r_n}, \cdots, T^{(n)}_0, S^{(n)}_1, \cdots, S^{(n)}_{2r_\nu}, C^{(n)}.\end{aligned}$$

Here $a_1, \cdots, a_n$ are distinct points in C. r_ν is the rank of the irregular singularity at a_ν. $T^{(\nu)}_{-r_\nu}, \cdots, T^{(\nu)}_0$ are the exponent matrices at a_ν. We assume that if $r_\nu\geqq 1$,

$$(2)\qquad t^{(\nu)}_{-r_\nu,\beta}\neq t^{(\nu)}_{-r_\nu,\beta}\qquad \text{for } \alpha\neq\beta,$$

where $T^{(\nu)}_{-r_\nu}=(t^{(\nu)}_{-r_\nu\alpha}\delta_{\alpha\beta})_{\alpha,\beta=1,\cdots,m}$. $S^{(\nu)}_1, \cdots, S^{(\nu)}_{2r_\nu}$ are the Stokes multipliers with respect to the sectors $S^{(\nu)}_{l,\delta}$ at a_ν (see (2.38) and (2.43) in [4]). $C^{(\nu)}$ is the connection matrix from a_ν to x_0. Note that $x=\infty$ is chosen to be a regular point for $Y(x_0, x)$. We should assume the following consistency conditions.

$$(3)\qquad \sum_{\nu=1}^{n}\sum_{\alpha=1}^{m} t^{(\nu)}_{0\alpha}=0,$$

$$(4)\qquad \begin{aligned}&(C^{(n)-1}e^{2\pi i T^{(1)}_0}S^{(n)-1}_{2r_n}\cdots S^{(n)-1}_1C^{(n)})\\ &\qquad\times\cdots\times(C^{(1)-1}e^{2\pi i T^{(1)}_0}S^{(1)-1}_{2r_1}\cdots S^{(1)-1}_1C^{(1)})=1.\end{aligned}$$

Under the above assumptions, we shall give a Neumann series for $Y(x_0, x)$ in (22), which is convergent if $T^{(\nu)}_{-j}$ and $S^{(\nu)}_l-1$ ($\nu=1, \cdots, n$; $j=0, 1, \cdots, r_\nu$; $l=1, \cdots, 2r_\nu$) are sufficiently small.

We also give expressions for the characteristic matrices $G^{(\nu,\mu)(l,k)}$ ($\nu, \mu=1, \cdots, n$; $l, k\geqq 1$) (see [6]) of the isomonodromy family. Since the characteristic matrices give rise to solutions to the non-linear deformation equations for the isomonodromy family, we thus obtain analytic expressions for these solutions. We refer the reader to [7]–[12] as for previous results on analytic expressions for solutions to Painlevé equations and their generalizations.