66. *A Note on the Large Sieve. IV*

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(Communicated Kunihiko KODAIRA, M. J. A., June 12, 1980)

1. The purpose of the present note is to show a hybrid of the multiplicative large sieve and the Rosser-Iwaniec linear sieve.

We retain most of the notations of our preceding paper [6], and in addition we introduce the following conventions: Let χ be a Dirichlet character, and put

$$S(A, z, \chi)=\sum_{\substack{n\in A\\(n,P(z))=1}}\chi(n)a_n,$$

where a_n are arbitrary complex numbers. We put also, for $\chi \pmod q$,

$$R_d(\chi)=\sum_{\substack{n\in A\\n\equiv 0\,(\mathrm{mod}\,d)}}\chi(n)-\varepsilon_\chi|\chi(d)|\frac{\delta(d)}{d}\prod_{p|q}\left(1-\frac{\delta(p)}{p}\right)X,$$

in which ε_χ is 1 if χ is principal, and 0 otherwise.

Then our hybrid sieve is

Theorem 1. *Let Δ be a finite set of primitive Dirichlet characters, and let M, N be arbitrary but $MN\geq z^2$. Then we have, as $z\to\infty$,*

$$\sum_{\chi\in\Delta}|S(A,z,\chi)|^2$$
$$\leq\left[XV(z)\left\{F\left(\frac{\log MN}{\log z}\right)+o(1)\right\}+O(E)\right]\sum_{\substack{n\in A\\(n,P(z))=1}}|a_n|^2,$$

where

$$E=\max_{\alpha,\beta}\max_{\psi\in\Delta}\sum_{\chi\in\Delta}\Big|\sum_{\substack{m<M\\n<N}}\alpha_m\beta_nR_{mn}(\chi\bar{\psi})\Big|,$$

$\{\alpha_m\}, \{\beta_n\}$ *being variable vectors such that* $|\alpha_m|\leq 1$, $|\beta_n|\leq 1$.

The proof which will be given in [7] is a direct application of Iwaniec's important idea [2] to the dual form

$$\sum_{\substack{n\in A\\(n,P(z))=1}}\Big|\sum_{\chi\in\Delta}\chi(n)b_\chi\Big|^2,$$

where b_χ are arbitrary complex numbers.

2. To illustrate the power of the above theorem we prove briefly the following result of the Brun-Tichmarsh type:

Theorem 2. *If $x\geq k^2Q^4\to\infty$, then we have*

$$\sum_{\substack{q\leq Q\\(q,k)=1}}\sum_{\chi\,(\mathrm{mod}\,q)}^{*}\Big|\sum_{\substack{p\equiv l\,(\mathrm{mod}\,k)\\p<x}}\chi(p)\Big|^2$$
$$\leq(2+o(1))x\left(\varphi(k)\log\left(\frac{x}{Q\sqrt{k}}\right)\right)^{-1}\pi(x;k,l),$$

where $\sum^*$ denotes a sum over primitive characters.