6. *On a Certain Integral Equation of Fredholm of the First Kind and a Related Singular Integral Equation*

By Yoshio HAYASHI*) and R. A. HURD**)

(Communicated by Kôsaku YOSIDA, M. J. A., Jan. 12, 1980)

1. It is the purpose of this paper to give an explicit formulation for the solution of an integral equation of Fredholm of the first kind

$$(1)\quad \int_l \left\{\frac{-1}{2\pi} ln\,|x-y| + C_0 + C_1(x-y)^2 + C_2(x-y)^2 ln\,|x-y|\right\}\tau(y)dy = g(x)$$

where $l=\bigcup_{j=1}^{\nu} l_j$ is the union of a finite number of bounded intervals $l_j=(a_j, b_j)$, $(a_j<b_j<a_{j+1}; j=1, 2, \cdots, \nu, a_{\nu+1}=\infty)$, C's are known complex valued constants, and $g(x)$ is a given, continuously differentiable function. The unknown function $\tau(x)$ is assumed to have a singularity of $O(1/\sqrt{x-c})$ at each of the end points $c=a_j$ and $c=b_j$ and otherwise is continuous.

In his previous paper [1], one of the authors showed that the Dirichlet problem for the Helmholtz equation for an open boundary l is equivalent to that of solving the integral equation

$$(2)\quad \int_l \psi(x, y)\tau(y)dy = g(x)$$

where $\psi(x, y)=(1/4i)H_0^{(2)}\ (k\,|x-y|)$ and $H_0^{(2)}$ is the second kind Hankel function of the zero-th order. If the "length" of l, or $(b_\nu - a_1)$, is such that $k^4(b_\nu - a_1)^4 = O(1)$ holds for a given "wave number" k, the kernel $\psi(x, y)$ of (2) is well approximated by that of (1), and (1) is an approximation of (2). If a solution of (1) is obtained, a higher order approximation to the solution of (2) is available by successive approximations.

On the other hand, after differentiation with respect to x, (1) is converted to the singular integral equation

$$(3)\quad \frac{1}{\pi i}\int_l \left\{\frac{1}{y-x} - A(y-x) - B(y-x) ln\,|x-y|\right\}\tau(y)dy = h(x)$$

where $A=2\pi(2C_1+C_2)$, $B=4\pi C_2$ and $h(x)=(2/i)(dg(x)/dx)$, and the integral is taken in the sense of Cauchy's principal value [1].

There are many works on singular integral equations [2], [3], however, to the best knowledge of the author, an equation like (3), whose kernel has a Cauchy type singularity and a log singularity simultaneously, has never been solved explicitly.

---

*) College of Science and Technology, Nihon University, Tokyo, and National Research Council of Canada.

**) Division of Electrical Engineering, National Research Council of Canada.