4. On the Initial Boundary Value Problem of the Linearized Boltzmann Equation in an Exterior Domain

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1. Problem and result. Let O be a bounded convex domain in $\boldsymbol{R}^n$ $(n\geq 3)$ with a smooth boundary and $\Omega=\boldsymbol{R}^n\backslash\bar{O}$. Put $Q=\Omega\times\boldsymbol{R}^n$ and $S^{\pm}=\{(x,\xi)\in\partial\Omega\times\boldsymbol{R}^n\,;\,n(x)\cdot\xi\gtrless 0\}$, where $n(x)$ is the inner normal of $\partial\Omega$ at x. For $u=u(t,x,\xi)$ which is related to the density of gas particles at time $t\geq 0$ and a point $x\in\Omega$ with a velocity $\xi\in\boldsymbol{R}^n$, our equation is described as follows;

(1.1) $\dfrac{\partial u}{\partial t}=-\sum_{j=1}^{n}\xi_j\dfrac{\partial u}{\partial x_j}-\nu(\xi)u+\int_{\boldsymbol{R}^n}K(\xi,\eta)u(t,x,\eta)d\eta.$

(1.2) $u|_{s^+}=C(u|_{s^-}).$

(1.3) $u|_{t=0}=u_0(x,\xi).$

Here C is a linear operator from a function space on S^- to the similar one on S^+. Our assumptions on the collision operator $L=\nu(\xi)-K$ are those of cut-off hard potentials.

(1.4) *$\nu(\xi)$ is continuous in ξ, depends only on $|\xi|$ and $\nu(\xi)\geq\nu_0>0$ for some constant ν_0.*

(1.5) *$K(\xi,\eta)=K(\eta,\xi)$ is real valued and continuous for $\xi\neq\eta$, $\int_{\boldsymbol{R}^n}|K(\xi,\eta)|^p\,d\eta<\infty$ for some p, $1<p<\infty$, $\int_{\boldsymbol{R}^n}|K(\xi,\eta)|\,(1+|\eta|)^{-\alpha}d\eta\leq d_\alpha(1+|\xi|)^{-\alpha-1}$ for any $\alpha\geq 0$.*

Moreover the operator L is non-negative self-adjoint in $L^2(\boldsymbol{R}^n)$ and has an isolated eigenvalue 0 with eigenfunctions $\{1,\xi_1,\cdots,\xi_n,|\xi|^2\}\times\exp\left(-\dfrac{1}{2}|\xi|^2\right)$. (Note that the operator K induced from the integral kernel $K(\xi,\eta)$ is a compact self-adjoint operator in $L^2(\boldsymbol{R}^n)$.)

As for the operator C we assume

(1.6) $\|C\|\leq 1$

as an operator from $L^2(S^-;\rho)$ to $L^2(S^+;\rho)$, where $\rho=\rho(x,\xi)=|n(x)\cdot\xi|$ and $L^2(S^{\pm};\rho)$ is the space of square integrable function on $S^{\pm}$ with respect to the measure $\rho(x,\xi)dS_xd\xi$.

We define the linearized Boltzmann operator B by

(1.7) $B=-\sum_{j=1}^{n}\xi_j\dfrac{\partial}{\partial x_j}-\nu(\xi)+K=-\xi\cdot\nabla_x-L$ with domain $D(B)$

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