27. Lévy's Functional Analysis in Terms of an Infinite Dimensional Brownian Motion. II

By Yoshihei HASEGAWA
Department of Mathematics, Nagoya Institute of Technology

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§1. **Introduction.** This note is a continuation of our previous note [2] and we shall use the terminologies in [2].

We shall consider various Dirichlet problems on the unit ball D_∞ in the space $\boldsymbol{E}$ defined in [2, §3]. In particular we shall establish a finite dimensional approximation theorem (Theorem 4.1) of Dirichlet solutions on D_∞ which may be regarded as a reformulation of Lévy's "la méthode du passage du fini à l'infini" (see [1, p. 307]).

§2. **Spherical Brownian motion** (continued from [2, §4]). The standard Gaussian white noise μ defined in [2, §2] can be easily extended to the measurable space $(S_\infty, \mathcal{S}_\infty)$ as follows:

$$\mu(A)=P^0(B(1,\omega)\in A) \qquad \text{for } A\in\mathcal{S}_\infty,$$

where $\{B(t)\}$ is the Brownian motion given in [2, §3].

Our first assertion is

Theorem 2.1. *Let $f(\xi)$ be a bounded, cylindrically measurable, O_1-continuous function on S_∞. Then we have*

$$\lim_{t\to\infty}\tilde{E}^\xi[f(\xi_t)]=\int f(\zeta)\mu(d\zeta) \qquad \textit{for any } \xi\in S_\infty,$$

and the white noise μ is the unique invariant probability measure of the spherical Brownian motion $\{\xi_t\}$.

Consequently we have the following contraction semi-group $\{T_t : t\geqslant 0\}$ ([5, Chap. IX]) on the complex Hilbert space $L^2(S_\infty,\mu)$:

$$T_t f(\xi)=\tilde{E}^\xi[f(\xi_t)] \quad (t\geqslant 0) \qquad \text{for } f\in L^2(S_\infty,\mu).$$

Now we have

Theorem 2.2. *The infinitesimal generator of $\{T_t\}$ is a self-adjoint operator with the pure-point spectrum $\{-n/2\,;\, n=0,1,2,\cdots\}$ and the eigenspace of the eigenvalue $-n/2$ is spanned by $\{\Xi_K\,;\,|K|=n\}$,* (see [2, §2] for definitions). *This infinitesimal generator agrees with the infinite dimensional Laplacian operator of Y. Umemura* (see [4]), *up to constant* $1/2$.

Next we shall see that the spherical Brownian motion is homogeneous under the group G of linear bimeasurable bijections g of $\boldsymbol{E}$ to $\boldsymbol{E}$ satisfying

$$\mu(g\cdot)=\mu(\cdot) \quad \text{and} \quad \|gx\|_\infty=\|x\|_\infty \quad \text{for } x\in\boldsymbol{E}.$$

Proposition 2.3. *For $g\in G$ and $A\in\mathcal{S}_\infty$, it holds that*