1. *A Note on Mikusiński's Operational Calculus*

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§1. **Introduction.** In [ii], one of the present authors gave a simplified derivation of Mikusiński's operational calculus [i] without appealing to Titchmarsh's theorem concerning the vanishing of the convolution of two continuous functions defined on $[0, \infty)$.

The purpose of the present note is to give a further simplification of [ii] to the effect that we can derive the operational calculus directly from the ring C_H in [ii] without introducing the ring C_P in [ii]. For the sake of convenience for the reader, we shall begin with the definition of the ring C_H.

§2. **The ring C_H.** We denote by C the totality of complex-valued continuous functions defined on $[0, \infty)$. We denote such a function by $\{f(t)\}$ or simply by f, while $f(t)$ means the value at t of the function f. For $f, g \in C$ and $\alpha, \beta \in \boldsymbol{K}$ (=the complex number field) we define

$$(1) \qquad \alpha f+\beta g=\{\alpha f(t)+\beta g(t)\} \quad \text{and} \quad fg=\left\{\int_0^t f(t-s)g(s)ds\right\}.$$

Then C is a commutative ring with respect to the above addition and multiplication over the coefficient field $\boldsymbol{K}$.

We shall denote by h (l in [i]) the constant function $\{1\} \in C$ so that we have

$$(2) \qquad h^n=\left\{\frac{t^{n-1}}{(n-1)!}\right\} \qquad (n=1, 2, \cdots),$$

and

$$(3) \qquad hf=\left\{\int_0^t f(s)ds\right\} \qquad \text{for } f \in C,$$

i.e. h behaves as an operation of integration. Then we have the following fairly trivial

Proposition 1. *For $k \in \boldsymbol{H}=\{k\,;\, k=h^n\ (n=1, 2, \cdots)\}$ and $f \in C$, the equation $kf=0$ implies that $f=0$, where 0 denotes $\{0\} \in C$.*

Therefore we can construct the commutative ring C_H of fractions:

$$(4) \qquad C_H=\left\{\frac{f}{k}\,;\ f \in C \text{ and } k \in \boldsymbol{H}\right\}$$

where the equality is defined by

$$(5) \qquad \frac{f}{k}=\frac{f'}{k'} \qquad \text{if and only if } k'f=kf',$$

and the addition and multiplication are defined through

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