20. On Excessive Functions

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It was pointed out by T. Watanabe [4, II] that Dynkin's criterion of excessiveness of a function f, is sometimes inconvenient for applications, because it requires two strong conditions:

1) the function f is finely continuous,

2) the function f is supermedian with respect to a very large family of sets.

As an alternative of Dynkin's criterion, Watanabe proved another criterion, in which he replaced the condition 1) with the stronger one, that f was lower semicontinuous, while condition 2) was weakened by considering a family $\mathcal{U}$ that had to be only a base. Furthermore it was conjectured that in this criterion the lower semicontinuity of f can be replaced by a weaker continuity condition stated in terms of $\mathcal{U}$.

Here we give a positive answer to this conjecture, in the case of an instantaneous state process. A version of this criterion is very useful in the case of a Markov process associated to an elliptic strongly degenerated differential operator [3].

Let E be a locally compact space with a countable open base and $\mathcal{E}$ the σ-algebra of Borel sets of E. Further let $(\Omega, \mathcal{M}, \mathcal{M}_t, X_t, \theta_t, P^x)$ be a standard process with state space $(E, \mathcal{E})$. For notations and definitions in the Markov process theory we refer to [1].

If A is a nearly Borel set, $f \in \mathcal{E}_+$ and $x \in E$ we denote $E^x[f(x_{T_{CA}})]$ by $H^A f(x)$.

Suppose that $\mathcal{U}$ is a family of nearly Borel sets such that for each point $x \in E$ and each neighbourhood V of x there exists $U \in \mathcal{U}$, $x \in \mathring{U}$, $U \subset V$. For any $x \in E$ the family $\mathcal{U}(x) = \{U \in \mathcal{U} / x \in \mathring{U}\}$ becomes a directed set under the order relation "$U_1 \leqslant U_2$ if $U_2 \subset \mathring{U}_1$".

Theorem. *If $s : E \to \bar{R}_+$ is an universally measurable function such that:*

(a) $H^U s \leqslant s$ *for any* $U \in \mathcal{U}$,

(b) $s(x) = \lim_{U \in \mathcal{U}(x)} H^U s(x)$ *for any* $x \in E$,

then s is excessive.

Proof. We consider a metric d on E and for each fixed $n \in N$,

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