2. *Studies on Holonomic Quantum Fields. XI*

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This paper is a direct continuation of our previous work [2]. We retain the same notations as in [2] without mentioning further.

1. In the present case of 2-dimensional Weyl equation, the orthogonal transformation $T[A]$ is the multiplication by $M(t)=M[A](t)$ where we have set $t=-x^{-}$. It is natural to ask if we can choose $Y_{\pm}$ and $Z_{\pm}$ to be multiplications by functions, say $Y_{\pm}(t)$ and $Z_{\pm}(t)$, respectively. The conditions (2) then require that $Y_{+}(t)$ and $Z_{+}(t)$ (resp. $Y_{-}(t)$ and $Z_{-}(t)$) are holomorphic in the upper (resp. the lower) half complex t-plane. This is the celebrated Riemann-Hilbert problem [1], [3].

Noting that $\lim_{|t|\to\infty} M(t)=1$, we can normalize $Y_{\pm}(t), Z_{\pm}(t)$ so that $\lim_{|t|\to\infty} Y_{\pm}(t)=\lim_{|t|\to\infty} Z_{\pm}(t)=1$. Then the unique solution is given by

$$X(t)=\sum_{n=0}^{\infty}(-)^{n}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}dt_1\cdots dt_n c_n(t_1,\cdots,t_n;t)(M(t_1)-1)\cdots(M(t_n)-1), \tag{21}$$

$$c_n(t_1,\cdots,t_n;t)=\begin{cases}\dfrac{1}{2\pi}\dfrac{-i}{t_1-t_2-i0}\cdots\dfrac{1}{2\pi}\dfrac{-i}{t_n-t-i0} & \text{for } X=Y_{+},\\[2ex] \dfrac{1}{2\pi}\dfrac{-i}{t-t_1-i0}\cdots\dfrac{1}{2\pi}\dfrac{-i}{t_{n-1}-t_n-i0} & \text{for } X=Y_{-}^{-1},\\[2ex] \dfrac{1}{2\pi}\dfrac{i}{t-t_1+i0}\cdots\dfrac{1}{2\pi}\dfrac{i}{t_{n-1}-t_n+i0} & \text{for } X=Z_{+}^{-1},\\[2ex] \dfrac{1}{2\pi}\dfrac{i}{t_1-t_2+i0}\cdots\dfrac{1}{2\pi}\dfrac{i}{t_n-t+i0} & \text{for } X=Z_{-}.\end{cases}$$

The kernel $\Phi(t,t')$ of $\Phi[T]$ in (3) reduces to

$$\Phi(t,t')=\frac{1}{2\pi i}\frac{1}{t-t'}(Y_{-}(t)^{-1}Y_{+}(t')-Z_{+}(t)^{-1}Z_{-}(t')). \tag{22}$$

In particular, we have

$$\begin{aligned}\Phi(t,t)&=\frac{1}{2\pi i}\left(\frac{dY_{-}(t)^{-1}}{dt}Y_{+}(t)-\frac{dZ_{+}(t)^{-1}}{dt}Z_{-}(t)\right),\\ &=\frac{-1}{2\pi i}\left(Y_{-}(t)^{-1}\frac{dY_{+}(t)}{dt}-Z_{+}(t)^{-1}\frac{dZ_{-}(t)}{dt}\right).\end{aligned} \tag{23}$$

Then from (7) we have the following

Theorem 4. *$\tau[T]$ is characterized by*