95. *Some Lie Algebras of Vector Fields on Foliated Manifolds and their Derivation Algebras*

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1. We want to define some structures on foliated manifolds and Lie algebras of vector fields associated with the structures, and determine their derivation algebras. We have two directions: One is to consider structures on leaves; the other on transversals to leaves. In this article we treat only the former (see [2] for details and proofs), and for the latter we will discuss elsewhere.

Let M be a $(p+q)$-dimensional smooth manifold, and $\mathcal{F}$ a codimension q foliation on M. Denote by $\mathcal{L}(M,\mathcal{F})$ the Lie algebra of all leaf-tangent vector fields on $(M,\mathcal{F})$, and by $\Omega(M)$ the exterior algebra of all differential forms on M, and define its differential ideal $\mathcal{J}(M,\mathcal{F})$ as

$$\begin{aligned}\mathcal{J}(M,\mathcal{F})&=\{\alpha\in\Omega(M)\,;\,\alpha(X_1,X_2,\cdots)=0 \text{ for } X_i\in\mathcal{L}(M,\mathcal{F})\}\\&=\{\alpha\in\Omega(M)\,;\,\iota_L^*\alpha=0 \text{ for every leaf } L \text{ of } \mathcal{F}\},\end{aligned}$$

where ι_L is the inclusion mapping of L in M. Then $\mathcal{J}(M,\mathcal{F})$ is L_X-stable for any $X\in\mathcal{L}(M,\mathcal{F})$, where L_X means the Lie derivative.

A p-form τ on M is called a partially unimodular structure on $(M,\mathcal{F})$, if $\iota_L^*\tau\neq 0$ for every leaf L of $\mathcal{F}$, that is, $\iota_L^*\tau$ is a volume form on L. Then τ is partially closed, that is, $d\tau\in\mathcal{J}(M,\mathcal{F})$.

Let $p=2n$. A 2-form ω on M is called a partially symplectic structure on $(M,\mathcal{F})$, if ω is partially closed and $\iota_L^*\omega$ is of rank $2n$ for every leaf L of $\mathcal{F}$.

Let $p=2n+1$. A 1-form θ on M is called a partially contact structure on $(M,\mathcal{F})$, if $(\iota_L^*\theta)\wedge(\iota_L^*d\theta)^n\neq 0$ for every leaf L of $\mathcal{F}$.

We can get normal forms of these partially classical structures on $(M,\mathcal{F})$ as follows; for suitable distinguished coordinates $(v_1,\cdots,v_p, w_1,\cdots,w_q)$

$$\tau\equiv dv_1\wedge\cdots\wedge dv_p,\quad \omega\equiv\sum_{i=1}^n dv_i\wedge dv_{i+n},\quad \theta\equiv dv_{2n+1}-\sum_{i=1}^n v_{i+n}dv_i \pmod{\mathcal{J}(M,\mathcal{F})}.$$

2. Let τ be a partially unimodular structure on $(M,\mathcal{F})$. A vector field $X\in\mathcal{L}(M,\mathcal{F})$ is called partially conformally unimodular, if $L_X\tau$ is congruent to $\phi\tau$ modulo $\mathcal{J}(M,\mathcal{F})$ for some function $\phi\in C^\infty(M)^{\mathcal{F}}$, where $C^\infty(M)^{\mathcal{F}}$ is the space of smooth functions on M which are constant on each leaves of $\mathcal{F}$. Moreover, if the function ϕ is zero, X is called partially unimodular. Then we get two natural Lie subalgebras of