94. *The Kodaira Dimension of Certain Fiber Spaces*

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In this paper we shall sketch a proof of the following theorem. Details will appear elsewhere.

Theorem. *Let $f: X \to Y$ be a morphism of non-singular projective algebraic varieties defined over the complex number field C with a general fiber F. We assume that F is irreducible and satisfies one of the following three conditions*:

(1) $\dim F = 1$,

(2) $\dim F = 2$ *and* $\kappa(F) \neq 2$,

(3) *F is an abelian variety.*

Then $\kappa(X) \geqslant \kappa(Y) + \kappa(F)$. *Moreover, if* $\kappa(F) = 0$, *then* $\kappa(X/Y) \geqslant \mathrm{Var}\,(f)$, *where* κ *and* Var *denote the Kodaira dimension and the variation, respectively* (cf. [3] and [7]).

In the cases (1) and (3) the above theorem was proved in [6] and [5], respectively. But our proof is different and does not use "good" compactifications of moduli spaces.

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Step 1. We assume that F is a $K3$ surface.

The period domain D of F is a bounded domain and its arithmetic quotient D/Γ has a Baily-Borel compactification $\overline{D/\Gamma}$ (cf. [1]). Of course the latter has nothing to do with the singular fibers of a family of F's. Let L be a free Z-module of rank 21 with a base $\{e_1, \cdots, e_{21}\}$ and with a non-degenerate inner product defined by $e_1e_{21} = e_2e_{20} = 1$, $e_3^2 = e_4^2 = \cdots = e_{19}^2 = -1$ and $e_ie_j = 0$, otherwise. Let Y_0 be a Zariski open subset of Y such that f is smooth on $X_0 = f^{-1}(Y_0)$. Let $F = F_y = f^{-1}(y)$ for a $y \in Y_0$. The polarization of X defines a homology class h on F_y and $H_2(F_y, C)/\langle h \rangle$ has a standard lattice isomorphic to L. A point p of D defines a 1-dimensional subspace in $\mathrm{Hom}\,(L, C)$. Let ω_p be its element such that $\omega_p(e_{21}) = 1$. If F and some Z-base of $H_2(F, Z)$ correspond to p, then ω_p defines a holomorphic 2-form on F, which we denote again by ω_p. If $q = \gamma p$ for some $\gamma \in \Gamma$, then $\omega_q = c\omega_p$, where $c = c(p)$ is an automorphic factor such that c^{19} is equal to the functional determinant. Thus each automorphic form $a(p)$ of weight k defines a