92. *The Invertibility Problem on Amphicheiral Excellent Knots**)

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The invertibility problem of knots is an old problem in knot theory. Although specific examples of non-invertible knots are obtained by H. F. Trotter [12] and W. Whitten [14], any reasonable invertibility invariants for testing examples are not known. Following R. Riley [8], we call a tame knot k in a 3-sphere S^3 *excellent* when S^3-k has a hyperbolic structure, i.e., a complete Riemannian metric of constant negative curvature with finite volume. By Thurston's existence theorem [11] of a hyperbolic structure, we see that many knots are excellent. In this paper we shall present an invertibility invariant for amphicheiral excellent knots. This invariant is enough to make a complete list of prime knots up to 10 crossings which are non-invertible and amphicheiral. Let $\langle t\rangle$ be an infinite cyclic group with a generator t and $Z\langle t\rangle$ be its group ring. Let f_1 and f_2 be in $Z\langle t\rangle$. By $f_1 \doteq f_2$ (or $f_1 \doteq_2 f_2$) we mean that f_1 and f_2 (or the Z_2-reductions of f_1 and f_2) are equal up to units of $Z\langle t\rangle$ (or $Z_2\langle t\rangle$). Let $k(t)$ be the Alexander polynomial ($\in Z\langle t\rangle$) of a tame knot k in S^3. Let $p_\lambda(t)=(t^\lambda-1)/(t-1)$ for any integer $\lambda>0$.

Theorem 1. *Let k be an excellent knot. If k is negative-amphicheiral, then* (1) $k(t^2)\doteq f(t)f(-t)$ *for* $f(t)\in Z\langle t\rangle$ *with* $f(-t)\doteq f(t^{-1})$ *and* $|f(1)|=1$. *If k is positive-amphicheiral, then* (2) *either* $k(t)\doteq f(t)^2$ *for* $f(t)\in Z\langle t\rangle$ *with* $f(t)\doteq f(t^{-1})$ *and* $|f(1)|=1$, *or there exist positive integers* n, λ *with* λ *odd such that* $k(t)\doteq f(t)^2 f_0(t)f_1(t)\cdots f_{n-1}(t)$ *for* $f(t), f_i(t)\in Z\langle t\rangle$ *with* $f(t)\doteq f(t^{-1})$, $f_i(t)\doteq f_i(t^{-1})$, $|f(1)|=|f_i(1)|=1$ *and* $f_i(t)\doteq_2 f(t)^{2^i+1}p_\lambda(t)^{2^i}$, $i=0,1,\cdots,n-1$. *If k is invertible and amphicheiral, then $k(t)$ satisfies both* (1) *and* (2).

Let h denote a piecewise-linear auto-homeomorphism of S^3 with $h(k)=k$. Then k is (*periodically* or *strongly*, resp.) *amphicheiral* if there is an orientation-reversing h (or finite order or of order 2, resp.); more precisely, k is (*periodically* or *strongly*, resp.) *positive-* or *negative-amphicheiral* according to whether $h|k$ is orientation-preserving or -reversing. k is (*strongly*) *invertible* if there is an orientation-preserving h (of order 2) such that $h|k$ is orientation-reversing.

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