86. On the Smoothness of Infinitely Divisible Distributions Corresponding to Some Ordinary Differential Equations

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1. Introduction. In the course of the investigation of the limit theorems of the decomposable Galton-Watson processes, the author [1] has found a class of the infinitely divisible distributions closely related to the following Riccati equations.

Let

$$\phi(t)=\sum_{n=0}^{\infty} a_n t^n,\ B>0 \quad \text{and} \quad m\geqq 0 \tag{1.1}$$

be given. We assume that every $a_n\geqq 0$ and $\phi(t)$ converges for all t. Let $\psi(t,\lambda)$, $t\geqq 0$, be the solution of

$$\frac{d}{dt}\psi(t,\lambda)=-B\psi(t,\lambda)^2+\phi(t)\lambda, \qquad \psi(0,\lambda)=m\lambda, \tag{1.2}$$

with $\lambda\geqq 0$ being a parameter.

Then we have

Theorem 1. (i) *For each $t>0$, there exists a probability measure P_t on $[0,\infty)$ such that*

$$\int_0^\infty e^{-\lambda x}P_t(dx)=\exp\left\{-\int_0^t \psi(s,\lambda)ds\right\}. \tag{1.3}$$

(ii) *P_t is infinitely divisible.*

(iii) *The Lévy measure n_t of P_t has the finite moments of all order.*

The probabilistic proof of (i) will be given in a forthcoming paper [1]. An alternative proof, which can be applied to more general equations, was given by T. Watanabe [2]. If we assume (i), (ii) is easily seen from $a\psi(t,\lambda;\phi,B,m)=\psi(t,\lambda;a\phi,a^{-1}B,am)$ for any $a>0$. (iii) follows from the fact that $\psi(t,\lambda)$ is C^∞ at $\lambda=0$.

The purpose of this paper is to show the following

Theorem 2. *Suppose that $\sum_{n=0}^{\infty} a_n>0$. Then there exists $d(t)>0$ such that*

$$\left|\int_0^\infty e^{i\lambda x}P_t(dx)\right|\leqq \exp\{-d(t)\sqrt{|\lambda|}\}, \tag{1.4}$$

for all sufficiently large $|\lambda|$. Therefore P_t is absolutely continuous with respect to the Lebesgue measure and the density belongs to $C^\infty(\boldsymbol{R})$.