# 8. *L-Equivalence Classes of Submanifolds in Complex Projective Spaces*

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**1. Introduction.** In a celebrated paper [7], Thom has developed a classification theory of submanifolds of a compact differentiable manifold $M$ roughly as follows. Two oriented submanifolds $N_1$ and $N_2$ of $M$ of codimension $p$ are said to be *L-equivalent* if they are oriented cobordant in $M \times I$. Let $\mathcal{L}_p(M)$ be the set of $L$-equivalence classes of submanifolds of $M$ of codimenision $p$. Then by making use of the transversality theorem, he has established a bijection

$$\pi : \mathcal{L}_p(M) \xrightarrow{\sim} [M, MSO(p)]$$

where the right hand side stands for the set of homotopy classes of maps from $M$ to $MSO(p)$, the Thom space for the group $SO(p)$. The correspondence is given by the so-called Pontrjagin-Thom map $\pi_N : M \to MSO(p)$, which is defined for every oriented submanifold $N$ of $M$ of codimension $p$. If we consider only those submanifolds of $M$ with complex normal bundles, we still have a bijection

$$\pi : \mathcal{L}_p^C(M) \xrightarrow{\sim} [M, MU(p)]$$

where $\mathcal{L}_p^C(M)$ is the set of (suitably modified) $L$-equivalence classes of submanifolds of codimension $2p$ with complex normal bundles of $M$ and $MU(p)$ is the Thom space for the group $U(p)$. Now assume that $M$ is an $n$-dimensional compact complex manifold. Then a natural question arises:

*Question.* Which element of $\mathcal{L}_p^C(M)$ or $\mathcal{L}_{2p}(M)$ can be represented by a *complex* submanifold of $M$?

If $M$ is a compact Kähler manifold, then there are some obvious conditions for an element in $\mathcal{L}_p^C(M)$ to be represented by a complex submanifold $N$ of codimension $p$ coming from the facts that the Poincaré dual of $N$ is a non zero element of $H^{p,p}(M)$ and also, under the Gysin homomorphism, the Chern classes of the normal bundle of $N$ go to the set of cohomology classes of type $(q, q)$ in the Hodge decomposition of the complex cohomologies of $M$. In this note we formulate a general condition other than the above and show that it is actually satisfied for a particular case when the ambiant manifold is the complex projective

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