73. *Irreducible Characters of p-Solvable Groups*

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1. Let G be a finite group and p a prime number. Let B be a p-block of G with a defect group D and b a p-block of $N_G(D)$ with $b^G=B$. It is conjectured in [1] that the number of irreducible complex characters in B of height 0 equals the number of those in b. In this note we shall show that this conjecture holds for p-solvable groups. A complete proof will be given elsewhere.

2. For a finite group G let $\mathrm{Ch}(G)$ (resp. $\mathrm{Irr}(G)$) denote the set of all characters (resp. irreducible characters) of G. If K is a normal subgroup of G and θ is an irreducible character of K, then we put $\mathrm{Irr}(G|\theta)=\{\chi\in\mathrm{Irr}(G)\,|\,(\chi_K,\theta)\neq 0\}$ and denote the set of all sums of elements in $\mathrm{Irr}(G|\theta)$ by $Ch(G|\theta)$. If B is a p-block of G, let $\mathrm{Irr}(B)$ be the set of irreducible characters of G in B.

The following theorem by Fong plays an important role in this note. We describe it using notation in a book of Isaacs [5, § 11].

Theorem (Fong [3]). *Let G be a finite group, K a normal p'-subgroup of G and $\theta\in\mathrm{Irr}(K)$. If θ is G-invariant, then there are a finite group $\hat{G}$, its cyclic central subgroup $\hat{K}$ and $\hat{\theta}\in\mathrm{Irr}(\hat{K})$ such that the following hold:*

(1) *there is an isomorphism $\tau: G/K\cong\hat{G}/\hat{K}$,*

(2) *for $K\subseteq H\subseteq G$ let $\hat{H}$ denote the inverse image in $\hat{G}$ of $\tau(H/K)$. For such subgroup H, there is a map $\sigma_H:\mathrm{Ch}(H|\theta)\to\mathrm{Ch}(\hat{H}|\hat{\theta})$ such that the following conditions hold for any $\chi,\psi\in\mathrm{Ch}(H/\theta)$:*

(a) $\sigma_H(\chi+\psi)=\sigma_H(\chi)+\sigma_H(\psi)$

(b) $(\chi,\psi)=(\sigma_H(\chi),\sigma_H(\psi))$

(c) $\sigma_G(\psi^G)=(\sigma_H(\psi))^{\hat{G}}$.

(3) *In (2), if b is a p-block of H such that $\mathrm{Irr}(b)\subseteq\mathrm{Irr}(H|\theta)$, then $\sigma_H(\mathrm{Irr}(b))=\mathrm{Irr}(\hat{b})$ for some p-block $\hat{b}$ of $\hat{H}$. Furthermore b and $\hat{b}$ have isomorphic defect groups and σ_H gives a $1-1$ height preserving correspondence between $\mathrm{Irr}(b)$ and $\mathrm{Irr}(\hat{b})$.*

The following result gives a connection between the above correspondence and Brauer's block correspondence.

Corollary. *In (3) in the above theorem assume that $DC_G(D)\subseteq H$*

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