72. *Homotopy Classification of Connected Sums of Sphere Bundles over Spheres. I*[*)]

By Hiroyasu ISHIMOTO

Department of Mathematics, Faculty of Science, Kanazawa University

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1. Statement of Results. Let A be a p-sphere bundle over a q-sphere $(p, q>1)$ which admits a cross-section, and consider the following diagram which is commutative up to sign.

$$\begin{array}{ccccc} & \overset{\partial}{\nearrow} & \pi_{q-1}(SO_p) & \xrightarrow{i_*} & \pi_{q-1}(SO_{p+1}) \\ \pi_q(S^p) & & \downarrow J & & \downarrow J \\ & \underset{P}{\searrow} & \pi_{p+q-1}(S^p) & \xrightarrow{E} & \pi_{p+q}(S^{p+1}) \end{array}$$

Here, $P=[\ ,\iota_p]$ means the Whitehead product with the orientation generator ι_p of $\pi_p(S^p)$. We denote the characteristic element of A by $\alpha(A)$. Let $\alpha(A)=i_*\xi$, $\xi \in \pi_{q-1}(SO_p)$. Then, $\{J\xi\} \in J\pi_{q-1}(SO_p)/P\pi_q(S^p)$ does not depend on the choice of ξ. We denote it by $\lambda(A)$ (James-Whitehead [4]).

Let A_i, $i=1, 2, \cdots, r$, be p-sphere bundles over q-spheres which admit cross-sections. It is understood that each A_i also denotes the total space of the bundle and has the differentiable structure induced from those of the fibre and the base space. $\#_{i=1}^r A_i$ means the connected sum $A_1 \# A_2 \# \cdots \# A_r$.

As an extension of James-Whitehead [4], we have the following

Theorem 1. *Let A_i, A'_i, $i=1, 2, \cdots, r$, be p-sphere bundles over q-spheres which admit cross-sections, and assume that $2p>q+1$, $q>1$, $p\neq q$. Then, the connected sums $\#_{i=1}^r A_i$, $\#_{i=1}^r A'_i$ are of the same homotopy type if and only if there exists a unimodular $(r\times r)$-matrix L of integer components such that*

$$\begin{pmatrix} \lambda(A'_1) \\ \vdots \\ \lambda(A'_r) \end{pmatrix} = L \begin{pmatrix} \lambda(A_1) \\ \vdots \\ \lambda(A_r) \end{pmatrix},$$

where the abelian group $J\pi_{q-1}(SO_p)/P\pi_q(S^p)$ is considered as a left Z-module.

Furthermore, we have the following

Theorem 2. *Even if $2p=q+1$, the conclusion of Theorem 1 holds also if p is odd and $p, q>1$.*

Let $p=q$. In this case, $\lambda(A_i)$, $\lambda(A'_i)$ belong to $J\pi_{p-1}(SO_p)/P\pi_p(S^p)$

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*) Dedicated to Professor A. Komatu for his 70th birthday.