70. On an Explicit Construction of Siegel Modular Forms of Genus 2

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1. In this note, we shall present an explicit correspondence from a pair of elliptic modular forms to a Siegel modular form of genus 2, which "preserves" Euler products, by means of theta series.

We denote by $\boldsymbol{H}$ the Hamilton quaternion algebra. For a ring A, let $A^\times$ denote the group of invertible elements of A. For a square matrix $M, \sigma(M)$ denotes the trace of M. For modular forms and Euler products associated with them, we shall use notation as is given in A. N. Andrianov [1] and G. Shimura [4].

2. Let D be a definite quaternion algebra over $\boldsymbol{Q}$ whose discriminant is d^2 and R be a maximal order of D. Let $D_A^\times$ denote the adelization of $D^\times$. For a prime l, we put $D_l=D\otimes_{\boldsymbol{Q}}\boldsymbol{Q}_l$ and $R_l=R\otimes_{\boldsymbol{Z}}\boldsymbol{Z}_l$ and let ι_l denote the canonical injection of $D_l^\times$ into $D_A^\times$. Set $K=\prod_l R_l^\times\times\boldsymbol{H}^\times$ and let $D_A^\times=\bigcup_{i=1}^{H} D^\times y_i K$ be a double coset decomposition of $D_A^\times$ such that the reduced norm of y_i $(1\leq i\leq H)$ is $1\in\boldsymbol{Q}_A^\times$. For $1\leq i,j\leq H$, define a lattice L_{ij} of D by $L_{ij}=D\cap y_i\left(\prod_l R_l\right)y_j^{-1}$ and put $R_i=L_{ii}$, $e_i=|R_i^\times|$. Let N, Tr and $*$ stand for the reduced norm, the reduced trace and the main involution of D respectively. Let H_n be the Siegel upper half space of genus n. Set

$$(1)\qquad \vartheta_{ij}(z)=\sum_{x\in L_{ij}}\exp(2\pi\sqrt{-1}N(x)z),\qquad z\in H_1,$$

$$(2)\qquad \tilde{\vartheta}_{ij}(z)=\sum_{(x,y)\in L_{ij}\oplus L_{ij}}\exp\left(2\pi\sqrt{-1}\sigma\left(\begin{pmatrix}N(x) & Tr(xy^*)/2\\ Tr(xy^*)/2 & N(y)\end{pmatrix}z\right)\right),\qquad z\in H_2.$$

Then ϑ_{ij} and $\tilde{\vartheta}_{ij}$ are Siegel modular forms of genera 1 and 2 respectively. The weight of them is 2 and the level of them is d. Let $S(R)$ denote the space of complex valued functions φ on $D_A^\times$ which satisfy that $\varphi(\gamma gk)=\varphi(g)$ for any $\gamma\in D^\times$, $k\in K$, $g\in D_A^\times$. For a prime $l\nmid d$, fixing a splitting $D_l\simeq M_2(\boldsymbol{Q}_l)$ such that R_l is mapped onto $M_2(\boldsymbol{Z}_l)$, we put

$$(T'(l)\varphi)(g)=\sum_{v=0}^{l-1}\varphi\left(g\cdot\iota_l\begin{pmatrix}l & v\\ 0 & 1\end{pmatrix}\right)+\varphi\left(g\cdot\iota_l\begin{pmatrix}1 & 0\\ 0 & l\end{pmatrix}\right).$$

For $\varphi,\varphi_1,\varphi_2\in S(R)$ and $1\leq i\leq H$, set

$$(3)\qquad f_i(\varphi)=\sum_{j=1}^{H}(\varphi(y_j)/e_j)\vartheta_{ij},$$