67. A Generalization of a Theorem of Marotto*)

By Kenichi SHIRAIWA**) and Masahiro KURATA***)

(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 12, 1979)

In 1975, Li and Yorke found the following theorem [3]. Let $f: I \to I$ be a continuous map of the compact interval I into itself. If f has a periodic point of minimal period three, then f exhibits chaotic behavior. This result is generalized by F. R. Marotto [4] in 1978 for the multi-dimensional case as follows. Let $f: \boldsymbol{R}^n \to \boldsymbol{R}^n$ be a differentiable map of the n-dimensional Euclidean space $\boldsymbol{R}^n$ $(n \geqq 1)$ into itself. If f has a snap-back repeller, then f exhibits chaotic behavior.

In this paper, we shall announce a generalization of the above theorem of Marotto. Our theorem can also be regarded as a generalization of the Smale's result [6] on the transversal homoclinic point of a diffeomorphism. A detailed proof will appear later.

§1. The main theorem. Let M be a smooth manifold of dimension n. Let $f: M \to M$ be a C^1-map, and let $z_0 \in M$ be a hyperbolic fixed point of f. We denote by $W^u_{\text{loc}}(z_0)$ (resp. $W^s_{\text{loc}}(z_0)$) a local unstable (resp. stable) manifold of f at z_0.

Main Theorem. *Let $f: M \to M$ be a C^1-map. Let $z_0 \in M$ be a hyperbolic fixed point of f. Assume the following three conditions.*

(1) $u = \dim W^u_{\text{loc}}(z_0) > 0$.

(2) *There exist a point $z_1 \in W^u_{\text{loc}}(z_0)$ $(z_1 \neq z_0)$ and a positive integer m such that $f^m(z_1) \in W^s_{\text{loc}}(z_0)$.*

(3) *There exists a u-dimensional disk B^u embedded in $W^u_{\text{loc}}(z_0)$ such that B^u is a neighborhood of z_1 in $W^u_{\text{loc}}(z_0)$, $f^m|B^u: B^u \to M$ is an embedding, and $f^m(B^u)$ intersects $W^s_{\text{loc}}(z_0)$ transversally at $f^m(z_1)$.*

Then the following conclusions hold.

(a) *There is a positive integer N such that there is a periodic point of f of minimal period p for any integer $p \geqq N$.*

(b) *There is an uncountable set S (called a scrambled set) in M satisfying the following conditions.*

(i) *S does not contain any periodic points.*

(ii) $f(S) \subset S$

(iii) $\limsup_{k \to \infty} d(f^k(x), f^k(y)) > 0$ *for any $x, y \in S$ $(x \neq y)$, where d is a compatible metric on M.*

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*) Dedicated to Professor A. Komatu on his 70th birthday.

**) Department of Mathematics, College of General Education, Nagoya University.

***) Department of Mathematics, Nagoya Institute of Technology.