66. *On the Existence of Solutions for Linearized Euler's Equation*

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1. Statement of results. Let Ω be a bounded domain in R^n with smooth boundary $\partial\Omega$ and ν be the unit exterior normal to $\partial\Omega$. We denote by H the real Hilbert space consisting of all the real vector fields u with coefficients in $L^2(\Omega)$ such that $\operatorname{div} u=0$ in Ω and $u\cdot\nu=0$ on $\partial\Omega$, and set $V=H\cap(H^1(\Omega))^n$. Denoting by P the orthogonal projection from $(L^2(\Omega))^n$ onto H, we consider the following initial value problem:

$$\text{(I.V.P.)}\qquad \begin{cases} \dfrac{du}{dt}+P(a,\operatorname{grad})u=f,\\ \qquad\qquad u(0)=u_0,\end{cases}$$

where $f=f(t)$ and $a=a(t)$ are given H-valued functions and u_0 is an element in H. (a,grad) denotes $\sum_{j=1}^n a^j(x,t)\partial/\partial x_j$. Our aim in this note is to establish the existence and uniqueness of the solution for (I.V.P.) under certain mild assumptions on data. As a byproduct, we have proved the essential self-adjointness of $iP(a,\operatorname{grad})$ as an operator on H when a does not depend on t. When $a=u$, (I.V.P.) is the initial value problem for Euler's equation of incompressible ideal fluids. However, we could not take a and u from the same function space (see Theorem 2 below). We note that nothing is known about the existence of global weak solutions for Euler's equation when $n\geqslant 3$.

Our method of proof is based on the "vanishing viscosity" argument for the following problem:

$$\text{(I.V.P.)}_\varepsilon\qquad \begin{cases} \dfrac{du}{dt}+\varepsilon Nu+P(a,\operatorname{grad})u=f,\\ \qquad\qquad u(0)=u_0,\end{cases}$$

where N denotes the Laplacian, $-\Delta$, acting on 1-forms with the Neumann boundary condition: $u\cdot\nu=0$, $(du)_{\mathrm{norm}}=0$ on $\partial\Omega$ which is associated with the bilinear form: $(du,dv)+(\delta u,\delta v)$, defined on $\{u\in(H^1(\Omega))^n;\ u\cdot\nu=0,\text{ on }\partial\Omega\}$, and $\varepsilon>0$ is a constant. Here we have denoted by d the exterior differentiation and by δ its formal adjoint. (Throughout this paper, vector fields and 1-forms are identified by means of Euclidean metric.) See [4] or [5] for the details of the Neumann problem for differential forms. It is easy to see that N

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