65. On a Construction of Multi-Soliton Solutions of the Pohlmeyer-Lund-Regge System and the Classical Massive Thirring Model

By Etsuro DATE
Department of Mathematics, Osaka University

(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 12, 1979)

1. This is a sequel to our previous paper [1]. We extend the method presented in [1] to the equation of the system of Pohlmeyer [11] and Lund-Regge [7]

$$(1)\qquad \begin{aligned}&\alpha_{\xi\eta}-\beta_{\xi}\beta_{\eta}\sin(\alpha/2)/2\cos^3(\alpha/2)+\sin\alpha=0,\\&\beta_{\xi\eta}+(\alpha_{\xi}\beta_{\eta}+\alpha_{\eta}\beta_{\xi})/\sin\alpha=0\end{aligned}$$

and the equation of the classical massive Thirring model

$$(2)\qquad \begin{aligned}&iu_{\eta}+2v+2u\,|v|^2=0,\\&iv_{\xi}+2u+2v\,|u|^2=0.\end{aligned}$$

As is well known, a typical class of nonlinear equations solvable by the inverse scattering method is the Zakharov-Shabat equations, which are a class of equations for $u_j(x, y, t)$, $v_k(x, y, t)$ expressed as

$$(3)\qquad [\textstyle\sum_{j=0}^{n} u_jD^j-\partial/\partial y,\ \ \sum_{k=0}^{m} v_kD^k-\partial/\partial t]=0,\quad D=\partial/\partial x.$$

The structures of this class are fairly well understood, especially in connection with algebraic geometry (see, for example, [4], [2], [9], [10]). However equations (1) and (2) are not included in this class. In [12], Zakharov and Mikhailov proposed that the sine-Gordon equation and equations (1), (2) are examples of the following class of equations. This class consists of equations of the relativistically invariant two-dimensional models in the classical field theories, which are expressed as the compatibility conditions of two linear differential equations

$$(4)\qquad i\Phi_{\xi}=U(\xi,\eta,\lambda)\Phi,\quad i\Phi_{\eta}=V(\xi,\eta,\lambda)\Phi,\quad \Phi=\Phi(\xi,\eta,\lambda),$$

U and V being matrix-valued rational functions of complex parameter λ with poles independent of (ξ, η). For equations in this class, Zakharov-Mikhailov gave a method of constructing new solutions when a particular solution is given (a kind of Bäcklund transformation). Therefore equations in this class are considered to have properties in common and to be investigated as a whole. By now, the structures of this class are less investigated compared with the class of equations (3), though for specific equations in this class the inverse scattering method has been applied, for example, for (1) by Lund [8] and Kulish [5] and for (2) by Kuznetsov-Mikhailov [6] and Kaup-Newell [3].

Here we construct multi-soliton solutions of (1) and (2), by char-