64. On a Nature of Convergence of Some Feynman Path Integrals. II

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§1. **Introduction.** In the previous note [8], we reported that under the assumptions (V-I) and (V-II) below, the Feynman path integral with the Lagrangean

$$L(t, x, \dot{x})=\frac{1}{2}|\dot{x}|^2-V(t, x)$$

converges in a very strong topology if the time interval is short. In the present note, we shall discuss the convergence of the Feynman path integral in the case the time interval is longer.

The potential function $V(t, x)$ is assumed to satisfy the following two assumptions;

(V-I) *$V(t, x)$ is a real valued function of (t, x). For any fixed $t\in \mathrm{R}$, $V(t, x)$ is of class C^∞ in x. $V(t, x)$ is a measurable function of $(t, x)\in \mathrm{R}\times \mathrm{R}^n$.*

(V-II) *For any multi-index α with its length $|\alpha|\geq 2$, the measurable function $M_\alpha(t)$ defined by*

$$M_\alpha(t)=\sup_{x\in \mathrm{R}^n}\left|\left(\frac{\partial}{\partial x}\right)^\alpha V(t, x)\right|+\sup_{|x|\leq 1}|V(t, x)|$$

is essentially bounded on any compact set of R.

We fix a large positive number K, say, $K=100(n+100)$. We let $T=\infty$ if $\operatorname{ess.sup}_{t\in\mathrm{R}} \sum_{2\leq|\alpha|\leq K} M_\alpha(t)<\infty$. Otherwise, we let T be any fixed finite positive number. We shall discuss everything in the time interval $(-T, T)$.

Let $S(t, s, x, y)$ be the classical action along the classical orbit starting from y at time s and reaching x at time t. We can prove that there exists a positive constant $\delta_1(T)$ such that $S(t, s, x, y)$ is uniquely defined for any x and $y\in \mathrm{R}^n$ if $|t-s|\leq \delta_1(T)$. See, [6], [7], and [8]. For $N=0, 1, 2, \cdots$, we shall consider the following integral transformation,

$$E^{(N)}(\lambda, t, s)\varphi(x)=\left(\frac{-\lambda}{2\pi(t-s)}\right)^{n/2}\int_{\mathrm{R}^n} a^{(N)}(\lambda, t, s, x, y)e^{\lambda S(t,s,x,y)}\varphi(y)dy, \tag{1}$$

where $\lambda=\sqrt{-1}\hbar^{-1}$, $\hbar$ being a small positive parameter (the Planck's constant), and the amplitude function is defined by (3) and (11) of [8]. Note that $E^{(0)}(\lambda, t, s)$ is the integral transformation that was used by