7. *On a Direct Method of Constructing Multi-Soliton Solutions*

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1. In this note we describe a direct method of constructing multi-soliton solutions of the nonlinear equations solvable by the inverse scattering method. In particular we consider the Zakharov-Shabat equations, the sine-Gordon equation and the equation of motion of the Toda Lattice.

2. Construction of simultaneous solutions. The above equations are expressed as the commutativity of two linear operators.

i) The Zakharov-Shabat equations [8]: they are a class of nonlinear differential equations for functions $u_j(x, y, t)$, $0 \leq j \leq n-2$, $v_k(x, y, t)$, $0 \leq k \leq m-2$ expressed as

$$[L-\partial/\partial y, M-\partial/\partial t]=0$$

where $L=\sum_{j=0}^{n} u_j D^j$, $M=\sum_{j=0}^{m} v_j D^j$, $D=\partial/\partial x$ and $u_n, u_{n-1}, v_m, v_{m-1}$ are constants. This class includes the Korteweg-de Vries equation, the Boussinesq equation, the two-dimensional Korteweg-de Vries equation as special cases.

ii) The sine-Gordon equation [1], [7]

$$u_{\xi\eta}+\sin u=0:$$

this equation is expressed as

$$[L, M]=0$$

where

$$L=\partial/\partial\xi-2^{-1}i\lambda\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}-2^{-1}iu_{\xi}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},$$

$$M=\partial/\partial\eta-2^{-1}i\lambda^{-1}\begin{pmatrix}0 & \exp(iu)\\ \exp(-iu) & 0\end{pmatrix}.$$

iii) The equation of motion of the Toda Lattice [6], [2], [4]

$$\partial Q_n/\partial t=P_n,$$
$$\partial P_n/\partial t=\exp(Q_{n-1}-Q_n)-\exp(Q_n-Q_{n+1}), \qquad n\in Z,$$

or

$$\partial a_n/\partial t=2a_n(b_{n+1}-b_n),$$
$$\partial b_n/\partial t=2a_n(a_n-a_{n-1})$$

where

$$a_n=4^{-1}\exp\{(Q_{n-1}-Q_n)/4\}, \qquad b_n=-2^{-1}P_n:$$

this equation is expressed as