6. On the Intersection Number of the Path of a Diffusion and Chains

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1. We are concerned with the following problem which was already considered by H. P. McKean [4] for the Brownian motion: in what manner does the path of a diffusion on a manifold wind around a fixed point or a hole asymptotically? For this purpose, we shall define a stochastic version of the intersection number. As is well-known, the usual intersection number can be represented by the integral of a differential double 1-form with singularity ([1]). Although the path of the diffusion is not smooth, we can define its intersection number with a chain by using the integral of the 1-form along the path defined in [2] (see also [3]). We then study the asymptotic behaviors of such random intersection numbers to get some solutions of the above mentioned problem.

2. Let M be a d-dimensional connected orientable Riemannian manifold with a Riemannian metric g and Δ be the Laplace-Beltrami operator corresponding to g. Let $L=\Delta/2+b$, where b is a C^∞ vector field on M. Consider the minimal diffusion process $X=(X_t, P_x)$ on M corresponding to L. For any continuous mapping $c:[0,t]\to M$, we denote by $c[0,t]$ the curve determined by c: $c[0,t]=\{c(s)\,;\,0\leqq s\leqq t\}$. We regard $c[0,t]$ as a singular 1-chain ([5]).

To define the intersection number, we prepare some notations. We principally use the notations of de Rham's book ([1]). Let $\bar{\mathcal{D}}$ be the space of square integrable currents. Set $\bar{\mathcal{D}}_1=\{T\in\bar{\mathcal{D}}\,;\,T$ is homologous to zero$\}$, $\bar{\mathcal{D}}_2=\{T\in\bar{\mathcal{D}}\,;\,T$ is cohomologous to zero$\}$ and $\mathcal{D}_3=\{T\in\bar{\mathcal{D}}\,;\,T$ is harmonic$\}$. Then $\bar{\mathcal{D}}=\bar{\mathcal{D}}_1+\bar{\mathcal{D}}_2+\mathcal{D}_3$. Let H_1, H_2, H_3 be the projections on $\bar{\mathcal{D}}_1, \bar{\mathcal{D}}_2, \mathcal{D}_3$ respectively. For any 1-current T which is continuous in mean at infinity, we define H_iT by $(H_iT,\phi)=(T,H_i\phi)$, $\phi\in C^\infty\cap\bar{\mathcal{D}}$, $i=1,2,3$. Then T can be decomposed uniquely as follows: $T=H_1T+H_2T+H_3T$. Denote by $h_i(x,y)$ the kernel of H_i, $i=1,2,3$. Let $e(x,y)=*_y h_1(x,y)$ be the adjoint form of h_1 (as 1-form of y). Then e is C^∞ if $x\neq y$. It is known that $e(x,y)$ can be written locally as follows. Let Δ be the Hodge-Kodaira's Laplacian acting on 1-forms. We can choose a domain U on which a fundamental solution $\gamma(x,y)$ for $\Delta\alpha=\beta$ exists. Let $\sigma(x,y)$ be a C^∞ function supported in $U\times U$ with (i) $0\leqq\sigma\leqq 1$, (ii)