5. *On Order Star-Finite and Closure-Preserving Covers*

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1. Introduction. R. Telgársky and Y. Yajima [6] have studied the structural properties of order star-finite covers and order locally finite covers. Moreover, in [6], they have proved the closure-preserving sum theorem for covering dimension; if a normal space X has a closure-preserving closed cover $\mathfrak{F}$ such that F is countably compact and $\dim F \leqq n$ for each $F \in \mathfrak{F}$, then $\dim X \leqq n$. This paper is a continuation of [6]. We first give a characterization of paracompact spaces in terms of order star-finiteness which is a generalization of star-finiteness. Secondly, using this result, we state a relation between order star-finite covers and order locally finite covers. Finally, we show that the closure-preserving sum theorem for large inductive dimension, as well as the above one, holds. All spaces are assumed to be Hausdorff spaces. N denotes the set of all natural numbers.

2. Order star-finite covers. A family $\{A_\lambda : \lambda \in \Lambda\}$ of subsets of a space X is said to be *order star-finite* [4] (*order locally finite* [1]), if one can introduce a well-ordering $<$ in the index set Λ such that for each $\lambda \in \Lambda$ the set A_λ meets at most finite many A_μ with $\mu < \lambda$ (the family $\{A_\mu : \mu < \lambda\}$ is locally finite at each point of A_λ). Then we may use, without loss of generality, the notation $\{A_\xi : \xi < \alpha\}$ instead of $\{A_\lambda : \lambda \in \Lambda\}$.

Proposition 1. *Every point-finite open cover of a collectionwise normal space X has an order star-finite open refinement.*

Proof. We modify the proof of E. Michael ([2], Theorem 2). Let $\mathfrak{U} = \{U_\lambda : \lambda \in \Lambda\}$ be a point-finite open cover of X. For $k \in N$, let Λ_k be the family of all $\gamma \subset \Lambda$ such that γ has exactly k elements. We shall construct a sequence $\{\mathfrak{V}_i : i \in N\}$ of families of open sets of X, where $\mathfrak{V}_i = \{V_\gamma : \gamma \in \Lambda_i\}$, satisfying the following conditions:

(1) $\mathrm{Cl}\, V_\gamma \subset \bigcap_{\lambda \in \gamma} U_\lambda$ for each $\gamma \in \Lambda_i$.

(2) $\mathfrak{V}_i$ is discrete for each $i \in N$.

(3) $\{\delta \in \bigcup_{j=1}^{i-1} \Lambda_j : V_\delta \cap V_\gamma \neq \varnothing\}$ is finite for each $\gamma \in \Lambda_i$.

(4) If $x \in X$ is an element of at most i elements of $\mathfrak{U}$, then $x \in \bigcup_{j=1}^{i} V_j$, where $V_j = \bigcup \{V_\gamma : \gamma \in \Lambda_j\}$.

Assume that $\mathfrak{V}_i = \{V_\gamma : \gamma \in \Lambda_i\}$ $(i = 1, \cdots, k)$ have been constructed to satisfy (1)–(4) for all $i \leqq k$. For each $\gamma \in \Lambda_{k+1}$, let $F_\gamma = (X \setminus \bigcup_{i=1}^{k} V_i) \cap (X \setminus \bigcup \{U_\lambda : \lambda \notin \gamma\})$. Then it follows from [2] that $\{F_\gamma : \gamma \in \Lambda_{k+1}\}$ is a