39. Perturbation of Domains and Green Kernels of Heat Equations. II

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§. **Introduction.** In our previous paper [2], the author gave Hadamard's variational formula of the Green kernels of heat equations with the Dirichlet boundary condition and gave the variational formula of the trace $T_r(t)$ of them. See also [3]. In this paper, in § 1, we study Hadamard's variational formula of the Green kernels of heat equations with the Neumann and the third boundary condition. In Theorem 2, we shall also give the variational formula of the trace $T_r(t)$ associated with them. In relation to Theorems 2 and 3, we study the dependence on domains of the eigenvalue of the Laplacian with the Neumann and the third boundary condition. In § 2, we give a rough sketch of the proof of Theorem 1. Details of the proof of Theorems 1 and 2 will be given elsewhere.

§ 1. **Hadamard's variational formulas.** Let Ω be a bounded domain in $\mathbf{R}^n$ with C^∞ boundary γ. Let $\rho(x)$ be a smooth function on γ and ν_x be the exterior unit normal vector at $x\in\gamma$. For sufficiently small $\varepsilon\geq 0$, let Ω_ε be the bounded domain whose boundary γ_ε is defined by

$$\gamma_\varepsilon=\{x+\varepsilon\rho(x)\nu_x\,;\,x\in\gamma\}.$$

Let $U_\varepsilon(x,y,t)$ be the Green kernel of heat equation with the third boundary condition, that is, $U_\varepsilon(x,y,t)$ has the following properties:

$$(1.1)_k \qquad \begin{cases} \left(\dfrac{\partial}{\partial t}-\Delta_x\right)U_\varepsilon(x,y,t)=0, & x,y\in\Omega_\varepsilon,\ t>0 \\ \lim\limits_{t\to+0} U_\varepsilon(x,y,t)=\delta(x-y), & x,y\in\Omega_\varepsilon \\ \left(\dfrac{\partial}{\partial\nu_x^\varepsilon}+k\right)U_\varepsilon(x,y,t)=0, & x\in\gamma_\varepsilon,\ y\in\Omega_\varepsilon,\ t>0, \end{cases}$$

where k is a fixed non-negative constant and $\dfrac{\partial}{\partial\nu_x^\varepsilon}$ denotes the derivative along the exterior normal direction at $x\in\gamma_\varepsilon$. We abbreviate $U_0(x,y,t)$ as $U(x,y,t)$. We give a notation. We fix $z\in\gamma$ and take an orthonormal basis $(z_1,\cdots,z_{n-1})$ on the tangent hyperplane at z. Then, we put

$$\langle\nabla_\gamma a(z),\nabla_\gamma b(z)\rangle=\sum_{j=1}^{n-1}\frac{\partial a}{\partial z_j}(z)\frac{\partial b}{\partial z_j}(z)$$