38. Note on Certain Nonlinear Evolution Equations of Second Order

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1. Introduction. In this note we consider nonlinear evolution equations of the form

(1.1) $$u''(t)+Au(t)+B(t)u'(t)=f(t), \qquad 0\leqq t\leqq T,$$

with initial conditions

(1.2) $$u(0)=u_0 \quad \text{and} \quad u'(0)=u_1,$$

$(u'(t)=du(t)/dt,\ u''(t)=d^2u(t)/dt^2)$, where A is a nonlinear operator and each $B(t)$ is a formally self-adjoint positive operator.

When $B(t)\equiv 0$, there are a great number of results on non-existence of global weak solutions of (1.1) (see e.g. Knops-Straughan [4] and the cited papers therein). However, as for the existence of a global weak solution for an abstract Cauchy problems (1.1) and (1.2), where A is a genuinely nonlinear operator, it seems that there are few results except for Tsutsumi's [8]. He obtained sufficient conditions for the global existince under the presence of the dissipative term $B(t)u'(t)$.

The purpose of the present note is to show the existence of a global weak solution of (1.1) and (1.2) satisfying a certain inequality of energy type. Especially, we intend to weaken the assumptions of Tsutsumi [8] so that the result can be applied to a wider class of nonlinear partial differential equations.

2. Assumptions and result. Let H be a real separable Hilbert space with inner product $(\cdot,\cdot)$ and norm $|\cdot|_H$. Let W be a second real separable Hilbert space with norm $|\cdot|_W$ and let V be a real separable reflexive Banach space with norm $|\cdot|_V$. Suppose that

$$V\subset W\subset H,$$

where each injection is dense and continuous. Furthermore, the injection of W into H is compact. As usual, we identify H with its own dual and denote by V^* and W^* the dual spaces of V and W, respectively. Then the following inclusion relation holds:

$$V\subset W\subset H\subset W^*\subset V^*.$$

The pairing between $x^*\in V^*$ (resp. $x^*\in W^*$) and $x\in V$ (resp. $x\in W$) is simply denoted by (x^*, x); if $x, x^*\in H$, this is the ordinary inner product in H.

Throughout this note we put the following assumptions on the nonlinear operator $A: V\to V^*$.