# 4. *Asymptotic Equivalence of Dynamical Systems*

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In [1] the author generalizes the notion of asymptotic equivalence and attempts to prove theorems which are related to results in [2, Chapter IX, Section 4]. Unfortunately, this definition of asymptotic equivalence is inadequate to guarantee the stated results. (This is not noted in the review of [1] in Mathematical Reviews, MR 45 #7211.) In this paper we present a counter example to two of the theorems in [1] and redefine "asymptotic equivalence" in such a manner as to validate the theorems to which we provide a counter example. In fact, stronger results, as would be expected by a more restrictive definition, are proved.

Throughout this paper $X$ will denote a locally compact metric space with metric $d$, $R$ the reals, and $R^+$ the nonnegative reals. If $M\subset X$ and $a>0$, then $K(M, a)$ will denote the set $\{x: d(x, M)<a\}$.

A dynamical system on $X$ is a continuous mapping $\pi: X\times R\to X$ such that

( i ) $\pi(x, 0)=x$ for all $x\in X$,

(ii) $\pi(\pi(x, s), t)=\pi(x, s+t)$ for all $x\in X$ and $s, t\in R$.

Let $\pi_i$ $(i=1, 2)$ be dynamical systems on $X$ and $x\in X$. Then $L_i(x)$ and $J_i(x)$ will denote the positive limit set of $x$ and the positive prolongational limit set of $x$, respectively, with respect to $\pi_i$. A compact subset $M$ of $X$ is called

( i ) a weak attractor of $\pi_i$, if there exists an $a>0$ such that $L_i(x)\cap M\neq\phi$ for every $x\in K(M, a)$,

( ii ) an attractor of $\pi_i$, if there exists an $a>0$ such that $\phi\neq L_i(x)\subset M$ for every $x\in K(M, a)$,

( iii ) a uniform attractor of $\pi_i$, if there exists an $a>0$ such that $\phi\neq J_i(x)\subset M$ for every $x\in K(M, a)$,

( iv ) stable with respect to $\pi_i$, if for any $a>0$ there exists $b>0$ such that $\pi_i(K(M, b), R^+)\subset K(M, a)$,

( v ) eventually stable with respect to $\pi_i$, if for any $a>0$ there exist $b>0$ and $T>0$ such that $\pi_i(K(M, b), [T, \infty))\subset K(M, a)$,

( vi ) weakly asymptotically stable with respect to $\pi_i$, if $M$ is eventually stable and a weak attractor with respect to $\pi_i$,

( vii ) asymptotically stable with respect to $\pi_i$, if $M$ is stable and an attractor with respect to $\pi_i$,