3. A Construction of the Fundamental Solution for the Schrödinger Equations

By Daisuke FUJIWARA
Department of Mathematics, University of Tokyo

(Communicated by Kôsaku YOSIDA, M. J. A., Jan. 16, 1979)

§ 1. **Introduction.** The aim of this note is to improve the results of [6], that is, to show that the main results of [6] hold even if we substitute the amplitude function $a(\lambda, t, s, x, y)$ of (10) in [6] by the constant function 1. We shall consider the Schrödinger equation

$$(1)\qquad \frac{\partial}{\lambda\partial t}u(t,x)+\frac{1}{2}\sum_{j=1}^{n}\left(\frac{\partial}{\lambda\partial x_j}\right)^2 u(t,x)+V(t,x)u(t,x)=0,\qquad (t,x)\in \mathrm{R}\times\mathrm{R}^n$$

and the initial condition

$$(2)\qquad u(s,x)=\varphi(x).$$

Here $\lambda=ih^{-1}$ is a pure imaginary parameter and h is a small parameter $0<h\leq 1$. The potential $V(t,x)$ is assumed to satisfy the following two conditions;

(V-I) $V(t,x)$ is real valued. For any fixed $t\in\mathrm{R}$, $V(t,x)$ is a C^∞ function of $x\in\mathrm{R}^n$. $V(t,x)$ is measurable in $(t,x)\in\mathrm{R}\times\mathrm{R}^n$.

(V-II) For any multi-index α with length $|\alpha|\geq 2$, the non-negative measurable function of t defined by

$$(3)\qquad M_\alpha(t)=\sup_{x\in\mathrm{R}^n}\left|\left(\frac{\partial}{\partial x}\right)^\alpha V(t,x)\right|+\sup_{|x|\leq 1}|V(t,x)|$$

is essentially bounded on every compact interval of R^1.

We fix $L\geq 10(m+n+10)$. We put $T=\infty$ if $\operatorname{ess.\,sup.}_{|\alpha|=L,t\in\mathrm{R}} M_\alpha(t)<\infty$. Otherwise we let T denote an arbitrarily fixed positive number. Every discussion will be made in the interval $(-T,T)$ throughout this paper.

We shall consider the integral transformation

$$(4)\qquad E(\lambda,t,s)\varphi(x)=\left(\frac{-\lambda}{2\pi(t-s)}\right)^{(1/2)n}\int_{\mathrm{R}^n}e^{\lambda S(t,s,x,y)}\varphi(y)dy,$$

where $S(t,s,x,y)$ is the classical action along the classical orbit starting the point y at the time s and reaching the point x at the time t. (If $|t-s|$ is small enough, such an orbit is uniquely determined. See Proposition 1 below.) The integral transformation (4) is exactly the same transformation as Feynman used in [3] and [4].

Let $[s,t]\subset(-T,T)$ be an arbitrary interval. Let

$$\Delta\,;\ s=t_0<t_1<t_2<\cdots<t_{L-1}<t_L=t$$

be an arbitrary subdivision of the interval $[s,t]$. We put