1. *A Simplified Derivation of Mikusiński's Operational Calculus*

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§1. **Introduction.** O. Heaviside introduced in 1899 an operational calculus [1] which he successfully applied to the integration of linear ordinary differential equations with constant coefficients. In his calculus occured certain operators whose interpretation as given by Heaviside and his successors is not only difficult to justify but also the range of validity of this calculus developed so far remains unclear. In 1949, J. Mikusiński inaugurated the theory [2] of convolution quotients by which he provided a clear and simple operational calculus well-suited for the purpose. His theory is based upon Titchmarsh's theorem concerning the vanishing of convolution of two continuous functions defined on $[0, \infty)$.

The purpose of the present paper is to show that we are able to simplify Mikusiński's operational calculus in such a way that we need not appeal to Titchmarsh's theorem at all. It is to be noted that the author has given in Okamoto [5] another approach which does not appeal to Titchmarsh's theorem.

§2. **The convolution ring $\mathfrak{C}$ and the ring $\mathfrak{C}_H$.** We denote by $\mathfrak{C}$ the totality of the complex number valued continuous functions f defined on $[0, \infty)$. In this paper, we write such functions by $\{f(t)\}$ or simply by f, while $f(t)$ means the value at t of the function f.

For $f, g \in \mathfrak{C}$, let the addition of two functions f and g be defined by

$$f+g=\{f(t)+g(t)\} \tag{1}$$

and the multiplication of f and g by the convolution:

$$fg=\left\{\int_0^t f(t-u)g(u)du\right\}. \tag{2}$$

Then we see that $\mathfrak{C}$ is a commutative ring with respect to this addition and this multiplication. We call it the *convolution ring*.

Throughout this paper, h will denote the *constant function* $\{1\}$. So we get

$$h^n=\left\{\frac{t^{n-1}}{(n-1)!}\right\} \qquad n=1, 2, 3, \cdots. \tag{3}$$

Furthermore, for any $f \in \mathfrak{C}$, we have

$$hf=\left\{\int_0^t f(u)du\right\}$$