83. *Completeness Criterion for Functions with Delay Defined over a Domain of Three Elements*

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§ 1. **Introduction.** Let $E_k=\{0, 1, \cdots, k-1\}$ for $k\geq 2$. A *k-valued function with delay* is defined to be a pair (f, d), where f is a usual k-valued logical (or switching) function, that is, a function from a cartesian product $E_k\times E_k\times\cdots\times E_k$ into E_k, and d is a nonnegative integer. It represents a certain switching element which performs its operation in d units of time delay. A set of such functions with delay is called *complete* if any k-valued function can be realized with some delay by composing the elements in the set "synchronously", i.e., so as to synchronize the delays caused at each part of the composition. A *spectrum* over E_k is an infinite sequence of sets of k-valued functions indexed by nonnegative integers, and it is used as an algebraic representation of a set of functions with delay. A spectrum is complete if and only if it is not included in any *maximal incomplete* spectrum, so that explicit determination of maximal spectra is important in order to effectively decide the completeness of an arbitrary given set of functions with delay. The above notions are developed as an extension of the classical theory on completeness for k-valued functions (e.g., [2], [5] and [6]).

V. B. Kudrjavcev [3] first considered the above completeness problem in the binary case $k=2$ (he called *completeness in the second sense*), and gave a criterion for completeness by explicitly determining all 11 classes of maximal spectra over E_2. For the case of general k, A. Nozaki [4] gave a criterion for completeness, and T. Hikita and A. Nozaki [1] gave a characterization of maximal spectra.

In this note, after introducing some preliminary results (§ 2), we reformulate the characterization of maximal spectra in [1] by using the notion of relation and classify them into three types named (A), (B) and (C) (§ 3). Then we apply this characterization to the three-valued case, and report the result of the explicit determination of all maximal spectra over E_3 (§ 4). The method is essentially in the same spirit as that in [6], and it is based on several lemmas on inclusion

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