73. A Residue Formula for Chern Classes Associated with Logarithmic Connections

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1. Let E be a holomorphic vector bundle over a complex manifold M, and D be a meromorphic connection of E. Under some assumptions below, we have a relation between the residues of D and the Chern classes of E. The purpose of this note is to state the relation. Full details and proofs will be published elsewhere.

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2. Let Z be the pole of D. We assume the following:

(H.1) *Z is normal crossing,*

(H.2) *Let $Z=\bigcup_{j\in N} Z_j$ be the decomposition of Z into irreducible components (N is a set of indices). Each Z_j is non-singular,*

(H.3) *D has a simple logarithmic pole along Z (see Deligne* [3]),

(H.4) *In the case when M is not compact, D is assumed to be integrable.*

Then the residue of D along Z_j, $\mathrm{Res}_j\, D$, is well-defined as a holomorphic section of $\mathrm{End}\,(E)|_{Z_j}$, the restriction of the endomorphism bundle of E onto Z_j [3].

Let $J=(j_1, j_2, \cdots, j_k)$ be an element of $N^k=N\times N\times\cdots\times N$ (k times). If among $j_1, j_2, \cdots, j_k$ there exists p different indices, say $j_1^*, j_2^*, \cdots, j_p^*$, put $J^*=\{j_1^*, j_2^*, \cdots, j_p^*\}$ and let a_m be the number of j_m^* appearing in J $(1\leqq m\leqq p)$.

Define $Z_{J^*}=\bigcap_{m=1}^{p} Z_{j_m^*}$. This is a submanifold of M of codimension p, not necessarily connected. Let $Z_{J^*}=\bigcup_i Z_{J^*}^{(i)}$ be the decomposition of Z_{J^*} into connected components.

It is known ([2], [3]).

Let $c_k\,(A_1, A_2, \cdots, A_k)$ be the completely polarized form of the k-th Chern polynomial $c_k(A)$ (A, A_j are matrices). Then for any element $J=(j_1, j_2, \cdots, j_k)$ in N^k,

$$c_k(\mathrm{Res}_{j_1} D, \mathrm{Res}_{j_2} D, \cdots, \mathrm{Res}_{j_k} D)$$

is constant on each component $Z_{J^*}^{(i)}$ by (H.4) or by the compactness of M when M is compact. Denote this value by $c_k\,(\mathrm{Res}_J\, D)^{(i)}$.

A submanifold W of M of codimension p determines an element