71. *Sylow Subgroups in a Pair of Locally Finite Groups*

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Introduction. Following Z. Goseki [2] we define a collection (A, B, f, g) as follows: Let A and B be groups. If there are homomorphisms f and g such that $\xrightarrow{g} A \xrightarrow{f} B \xrightarrow{g} A \xrightarrow{f}$ is exact, we say that the collection (A, B, f, g) is well defined. Suppose (C, D, f_1, g_1) is well defined where C and D are subgroups of A and B respectively. If $f_1 = f$ on C and $g_1 = g$ on D then we call (C, D, f_1, g_1) a subgroup of (A, B, f, g) and denote it by (C, D, f, g). If $C \lhd A$ and $D \lhd B$ then (C, D, f, g) is a normal subgroup of (A, B, f, g). Goseki [2] states that if (C, D, f, g) is a subgroup such that C is a Sylow p subgroup of A then D is a Sylow p subgroup of B.

We prove that this statement does not hold in general but does hold for a wide class Γ_p of groups which contains for example periodic soluble linear groups and FC groups (locally normal groups). π will always denote a set of primes and π' its complementary set.

The following fact is a direct consequence of Zorn's lemma. "Let G be any group. Then every π subgroup of G is contained in a maximal π subgroup of G". In particular G possesses a maximal π subgroup, we shall refer to such as S_π subgroups.

Definitions. 1) *A local system for a group G is a set Σ of subgroups such that every finite subset of G is contained in some member of Σ.*

2) *A group is locally finite if it has a local system consisting of finite subgroups.*

In this paper all groups will be locally finite and all local systems will consist of finite subgroups.

Following [8] we define an S_π subgroup to be good if it reduces into a local system.

Definition. *An S_π subgroup P of G is good with respect to a local system Σ if for each $X \in \Sigma$ we have that $P \cap X$ is an S_π subgroup of X. We say that P is good if there is some local system with respect to which it is good.*

It is not hard to prove ([8], Proposition 1.12) that

Proposition 1. *If N is a normal subgroup of G, P is an S_π subgroup of G which is good with respect to Σ and $P \cap X$ is a Hall π subgroup of X for each X then $P \cap N$ and PN/N are good S_π subgroups*