# 67. *Convergence and Approximation of Integral Solutions of Nonlinear Evolution Equations*

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**1. Introduction.** Let $X$ be a Banach space with norm $|\ |$. A subset $A$ of $X\times X$ is said to be $\omega$-accretive if $\tau(x_1-x_2, y_1-y_2)+\omega|x_1-x_2|$ is non-negative for every $[x_i, y_i]\in A$, $i=1,2$, where $\tau(x,y)=\inf_{\lambda>0}\lambda^{-1}(|x+\lambda y|-|x|)$ for $x, y\in X$.

Consider the following Cauchy problem

(1) $$du/dt+Au\ni 0,\quad 0\leqq t<T,\quad u(0)=x.$$

According to Bénilan [1], a continuous function $u:[0,T)\to X$ is called an integral solution of type $\omega$ (for simplicity an $\omega$-integral solution) of (1), if it satisfies $u(0)=x$ and

(2) $$e^{-\omega t}|u(t)-u|-e^{-\omega s}|u(s)-u|\leqq\int_s^t e^{-\omega\sigma}\,\tau(u(\sigma)-u,\ -v)d\sigma$$

for every $[u,v]\in A$ and $0\leqq s\leqq t<T$.

Concerning the existence of an $\omega$-integral solution of (1) in a general Banach space, sufficient conditions were given by Crandall and Liggett [4], Bénilan [1], Y. Kobayashi [5] and Pierre [8]. Some of them were then applied by Brezis and Pazy [3], Kurtz [6], Miyadera and Kobayashi [7] and others to obtain convergence and approximation theorems for the semigroups corresponding to $\omega$-integral solutions.

In this paper we deal with some problems of similar nature, but in a slightly different manner. Our method does not depend upon any theorem on generation of nonlinear semigroups. Instead, we make use of a necessary condition for an $X$-valued function to be an $\omega$-integral solution of (1) (Lemma 1). Assuming the existence of an $\omega$-integral solution $u(t)$ of (1), we estimate the error of it, the difference between $u(t)$ and its approximation throughout. Our results, the statements of which appear somewhat complicated, still include most of the results obtained by the previous authors.

**2. The main theorems.** Let $\{A_n\}_{n=1}^{\infty}$ be a sequence of subsets $\subset X\times X$. By $\mathrm{Lim}\,A_n\supset A$ we mean that $A_n$ converge to $A$ in the sense of Kurtz [6], that is, for every $[u,v]\in A$ there exist $[u_n,v_n]\in A_n$ such that $\lim(|u_n-u|+|v_n-v|)=0$.

We first study a relation between the convergence of $\omega_n$-integral solutions $u_n(t)$ of the Cauchy problems

$(1)_n$ $$du/dt+A_nu\ni 0,\quad 0\leqq t<T,\quad u(0)=x_n$$