66. *Studies on Holonomic Quantum Fields. IX*

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In this note we shall give a symplectic version of the 2-dimensional operator theory, previously expounded in the orthogonal case [2], [5], [6]. Of particular interest is the neutral theory discussed in § 4. Corresponding to the bose field $\varphi^F(a)$ [1], there arises a strongly interacting fermi field $\varphi^B(a)={}^t(\varphi^B_+(a), \varphi^B_-(a))$. These two fields $\varphi^F(a)$ and $\varphi^B(a)$ are shown to share the same S-matrix in common, and their τ-functions are related to each other through simple formulas (34), (36), (38)–(39) (cf. IV–(49) [2]).

We remark that the 1-dimensional Riemann-Hilbert problem [4], [8] is also dealt with in the symplectic framework.

We follow the notations used throughout this series [1]–[6].

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1. Let W be an N-dimensional complex vector space equipped with a skew-symmetric inner product $\langle\ ,\ \rangle$. Let $A(W)$ be the algebra generated by W with the defining relation $ww'-w'w=\langle w, w'\rangle$. Denote by $S(W)$ the symmetric tensor algebra over W. As in the orthogonal case [3], [7], the norm map

$$\text{(1)} \qquad \mathrm{Nr}: A(W) \xrightarrow{\sim} S(W)$$

and the expectation value $\langle a\rangle$ of $a \in A(W)$ are defined analogously, by specifying a bilinear form $(w, w') \to \langle ww'\rangle$ on W such that $\langle ww'\rangle - \langle w'w\rangle = \langle w, w'\rangle$ $(w, w' \in W)$.

Now let $v_1, \cdots, v_N$ be a basis of W, and set $K=(\langle v_\mu v_\nu\rangle)$, $H=(\langle v_\mu, v_\nu\rangle)=K-{}^tK$. Consider an element g of the form

$$\text{(2)} \qquad \mathrm{Nr}\,(g)=\langle g\rangle e^{\rho/2}, \qquad \rho=\sum_{\mu,\nu=1}^N R_{\mu\nu}v_\mu v_\nu = vR^tv$$

with $v=(v_1, \cdots, v_N)$. Contrary to the orthogonal case, $e^{\rho/2}$ no longer belongs to $S(W)$. So we let $R_{\mu\nu}=R_{\nu\mu} \in t\cdot \boldsymbol{C}[[t]]$, and regard g (resp. $e^{\rho/2}$) as an element of $A(W)[[t]]$ (resp. $S(W)[[t]]$), the formal power series ring with coefficients in $A(W)$ (resp. $S(W)$). The norm map (1) is uniquely extended there. (This formulation is due to T. Miwa.) Most of the formulas in the orthogonal case are valid for g of the form (2), if we replace ${}^tK$ by $-{}^tK$. We tabulate below formulas corresponding to (1.5.5)–(1.5.6), (1.5.7)–(1.5.8) and (1.4.6)–(1.4.7) of [7].

$$\text{(3)} \qquad \mathrm{Nr}\,(wg)=\Big(\sum_{\mu,\nu=1}^N v_\mu(1+R^tK)_{\mu\nu}c_\nu\Big)\cdot\langle g\rangle e^{\rho/2}$$