54. *Hyperbolic Nonwandering Sets without Dense Periodic Points*

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Let $f: M \to M$ be a C^∞ diffeomorphism of a closed C^∞ manifold M, and let $\Omega(f)$ be the nonwandering set of f. $\Omega(f)$ is hyperbolic if $\Omega(f)$ is compact and the restriction $T_{\Omega(f)}M$ of the tangent bundle TM of M on $\Omega(f)$ splits into the Whitney sum of Tf-invariant subbundles

$$T_{\Omega(f)}M = E^s \oplus E^u,$$

such that given a Riemannian metric on TM there are positive numbers c and $\lambda<1$ such that $|Tf^n v| < c\lambda^n |v|$, for $v \in E^s$ and $n>0$, and $|Tf^{-n}v| < c\lambda^n |v|$, for $v \in E^u$ and $n>0$. The following problem was suggested in [3].

Problem. If a nonwandering set $\Omega(f)$ is hyperbolic, are the periodic points dense in $\Omega(f)$?

Newhouse and Palis proved that the answer is affirmative when M is a two dimensional closed manifold ([1] and [2]).

In this paper we give the following

Theorem. *Suppose* $\dim M \geq 4$. *Then there is a diffeomorphism* $F: M \to M$ *such that the nonwandering set* $\Omega(F)$ *is hyperbolic but its periodic points are not dense in* $\Omega(F)$.

Construction. To simplify the construction, we assume $\dim M = 4$.

1. Denote $D = [-2, 6] \times [-1, 3] \subset R^2$. Let an embedding $f: D \to D$ satisfy the followings (Fig. 1). Suppose that real numbers $a_{-1}, \cdots, a_6$ satisfy

(1.1) $a_{-1} = -2 < -1 < a_0 = -a_1 < 0 < a_1 < 1 < a_2 < a_3 < a_4 < 4 < a_5 < 5 < a_6 = 6$, and the rectangle A_i $(i=0, \cdots, 6)$ is given by

$$A_i = \{(x, y) \in D \mid a_{i-1} \leq x \leq a_i\}.$$

Then f satisfies (1.2)–(1.5).

(1.2) $f|A_0, f|A_2$ and $f|A_6$ are contractions with three sinks $(-1, 0)$, $(1, 0)$ and $(5, 2)$,

(1.3) $f(A_4) \subset \operatorname{int} A_0$,

(1.4) $f|A_i: A_i \to f(A_i)$ $(i=1, 3, 5)$ maps A_i linearly onto a rectangle $f(A_i)$, expanding horizontally and contracting vertically. There are two hyperbolic fixed points, $(0, 0)$ and $(4, 2)$.

(1.5) There are numbers $\alpha>1$ and $0<\beta<1$ such that