53. *Reparametrization and Equicontinuous Flows*

By Jirō EGAWA
Faculty of General Education, Kobe University

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Let X be a topological space, and R denotes the set of real numbers. A continuous mapping $\pi: X \times R \to X$ is said to be *a dynamical system or a flow on (a phase space) X* if π satisfies the following two conditions:

(1) $\pi(x, 0)=x$ for $x \in X$,

(2) $\pi(\pi(x, t), s)=\pi(x, t+s)$ for $x \in X$ and $t, s \in R$.

$C_\pi(x)$ denotes the orbit of π through $x \in X$. In this paper we always assume that phase spaces of flows are compact and connected metric spaces, and that every flow admits no singular point ($x \in X$ is called *a singular point of π* if $C_\pi(x)=\{x\}$). A flow π is said to be *equicontinuous* if $\{\pi_t\}_{t \in R}$ forms an equicontinuous family of homeomorphism of X onto Y, where π_t is defined by $\pi_t(x)=\pi(x, t)$ for $x \in X$. Let π and ρ be flows on X and Y, respectively. A homeomorphism h of X onto Y is called *an isomorphism of π onto ρ* if $h(C_\pi(x))=C_\rho(h(x))$ for $x \in X$. In this case, it is known ([1]) that there exists a continuous function $\phi: X \times R \to R$, which is called the reparametrization for h, satisfying $h(\pi(x, t))=\rho(h(x), \phi(x, t))$ for $(x, t) \in X \times R$. We can easily verify the above reparametrization ϕ satisfies the following condition (A):

(A) $\phi(x, t+s)=\phi(\pi(x, t), s)+\phi(x, t)$ for $x \in X$ and $t, s \in R$.

Further, if the both flows are equicontinuous, then ϕ is uniformly continuous on $X \times R$ ([2]). In this paper we shall show the following

Theorem. *Let π be an equicontinuous flow on X, and let ϕ be a continuous function on $X \times R$ satisfying the property* (A). *If ϕ is uniformly continuous on $X \times R$, then there exist a real number α and a continuous function $\Phi: X \to R$ satisfying*

$$\phi(x, t)=-\Phi(\pi(x, t))+\Phi(x)+\alpha t \qquad \text{for } (x, t) \in X \times R.$$

To prove the theorem, we need several lemmas. Put $F_t(x) = \dfrac{\phi(x, t)}{t}$ for $(x, t) \in X \times [1, \infty)$.

Lemma 1. *$\{F_t\}_{t \geqq 1}$ is uniformly bounded and equicontinuous.*

Proof. Equicontinuity of $\{F_t\}$ follows from the uniform continuity of ϕ. By the property (A) we have

$$\phi(x, t)=\phi(\pi(x, t-1), 1)+\phi(x, t-1)$$