52. Best Possibility of an Integral Test for Sample Continuity of L_p-Processes ($p\geq 2$)

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§1. **Introduction.** Let $\{X(t,\omega);\ 0\leq t\leq 1,\ \omega\in\Omega\}$ be a real valued separable stochastic process defined on a probability space $(\Omega,\mathfrak{A},P)$. We concern a best possible integral test for sample continuity of all processes belonging to an indicated class. For this aim, we define for $p\geq 1$,

S_p = a collection of all separable stochastic processes $\{X(t,\omega);\ 0\leq t\leq 1,\ \omega\in\Omega\}$ up to equivalent class such that

$$(E[|X(t)|^p])^{1/p}=\|X(t)\|_p<+\infty \qquad \text{for all } 0\leq t\leq 1,$$

Σ = a collection of all continuous function on [0, 1] which are non-negative, non-decreasing and zero at the origin,

and for $\sigma\in\Sigma$

$$S_p(\sigma)=\{\{X(t)\}\in S_p;\ \|X(t)-X(s)\|_p\leq\sigma(|t-s|)\}.$$

Then the following integral test for sample continuity of all processes belonging to $S_p(\sigma)$ is known ([1]).

Theorem A. *If*

$$I_p(\sigma)=\int_{+0} h^{-(1+1/p)}\sigma(h)dh<+\infty,$$

then all processes belonging to $S_p(\sigma)$ have continuous sample paths with probability 1.

The converse statement is not true in general, but Hahn-Klass [2] have proved the following theorem using a rearrangement of σ in case of $p=2$.

Set $\bar{\sigma}(h)=\inf_{y\geq 1} y\sigma(h/y)$.

Theorem B. *All processes belonging to $S_2(\sigma)$ have continuous sample paths with probability* 1 *if and only if $I_2(\bar{\sigma})$ converges.*

In this paper, we establish some relation concerning about $\bar{\sigma}$ and extend Theorem B to $p\geq 2$ by just the analogous method as them.

§2. Set

$$\sigma_*(h)=\sup_{\{X(t)\}\in S_p(\sigma)}\ \sup_{\substack{0\leq s\leq h\\ 0\leq t\leq t+s\leq 1}}\|X(t+s)-X(t)\|_p,$$

and

$$\sigma^*(h)=\text{the largest sub-additive minorant of } \sigma,$$

that is, σ^* is characterized by the following:

(i) $\sigma^*\in\Sigma$,