51. On Some Unilateral Problem of Elliptic and Parabolic Type

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In this note we establish some regularity result for the parabolic unilateral problem

$$\partial u/\partial t+Lu\geqq f,\qquad u\geqq \Psi,$$
$$(\partial u/\partial t+Lu-f)(u-\Psi)=0$$

as well as some related result for the associated elliptic problem.

Let Ω be a not necessarily bounded domain of R^n with smooth boundary Γ. Let

$$a(u,v)=\int_\Omega\left(\sum_{i,j=1}^N a_{ij}\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_j}+\sum_{i=1}^N b_i\frac{\partial u}{\partial x_i}v+cuv\right)dx$$

be a bilinear form defined on the space $H^1(\Omega)\times H^1(\Omega)$ of real valued functions with real coefficients $a_{ij}\in B^1(\bar\Omega)$, $a_i\in B^1(\bar\Omega)$, $c\in L^\infty(\Omega)$, where $B^1(\bar\Omega)$ is the set of functions continuous and bounded in $\bar\Omega$ together with first derivatives. Assume that the matrix $\{a_{ij}(x)\}$ is uniformly positive definite in Ω and there exists a positive number α such that

$$c\geqq\alpha,\ c-\sum_{i=1}^N\partial b_i/\partial x_i\geqq\alpha\quad\text{a.e.}$$

Let

$$L=-\sum_{i,j=1}^N\frac{\partial}{\partial x_j}\left(a_{ij}\frac{\partial}{\partial x_i}\right)+\sum_{i=1}^N b_i\frac{\partial}{\partial x_i}+c$$

be the differential operator associated with the bilinear form $a(u,v)$. For $1\leqq p\leqq\infty$ we denote by L_p the realization of L in $L^p(\Omega)$ under the Dirichlet boundary condition (refer to [2] or [6] for this subject where Ω is assumed to be bounded). Let Ψ be a function defined in Ω.

(Ψ.1) For some p, $1<p<\infty$, $\Psi\in W^{2,p}(\Omega)$ and $\Psi|_\Gamma\leqq 0$.

(Ψ.2) $\Psi\in W^{1,1}(\Omega)$, $L\Psi\in L^1(\Omega)$ and $\Psi|_\Gamma\leqq 0$.

By M_p we denote the multivalued mapping defined by

$D(M_p)=\{u\in L^p(\Omega): u\geqq\Psi$ a.e. in $\Omega\}$,

$M_pu=\{g\in L^p(\Omega): g\leqq 0$ a.e. in Ω, $g(x)=0$ where $u(x)>\Psi(x)\}$.

When the assumption (Ψ.1) is satisfied, we define the operator A_p by $A_p=L_p+M_p$; when the assumption (Ψ.2) as well as (Ψ.1) for some $1<p<\infty$ is satisfied, we define the operator A_1 by $A_1=L_1+M_1$.

Proposition 1. *A_p and A_1 are m-accretive in $L^p(\Omega)$ and $L^1(\Omega)$ respectively and*