48. On the Normal Generation by a Line Bundle on an Abelian Variety

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Let k be an algebraically closed field of characteristic $p\geqq 0$, X an abelian variety over k, and L an ample invertible sheaf on X. In the previous paper [7], the author proved, unfortunately providing $p\neq 2,3$, that the embedded variety of X into the projective space $\boldsymbol{P}(\Gamma(L^3))$ by means of the global sections of L^3, is ideal-theoretically an intersection of cubics. But he has recently found that the method in it can be extended for $p=2$. Namely, it can be checked easily even for $p=2$ that the canonical map $\Gamma(L^2\otimes P_\alpha)\otimes\Gamma(L^2\otimes P_{-\alpha})\to\Gamma(L^4)$ is surjective for almost all α in $\hat{X}$, and which was the only obstacle to the proof of the fact for $p=2$ in [7]. Moreover, as mentioned in [7], the surjectivity of the canonical maps lead us easily to the main results given in [1], [5] and [6].

We start with the following Mumford's theta structure theorem.

Theorem (Mumford's theta structure theorem). *Let M be a non-degenerate invertible sheaf on X of index i, and*

$$1\longrightarrow \boldsymbol{G}_m\longrightarrow \mathcal{G}(M)\xrightarrow{j(M)} K(M)\longrightarrow 0$$

the theta group scheme of M, where $K(M)$ is the scheme-theoretic kernel of the homomorphism $\phi_M: X\to\hat{X}$ defined by $x\mapsto T_x^*M\otimes M^{-1}$. Then the canonical action U of $\mathcal{G}(M)$ on $H^i(M)=H^i(X,M)$ is the unique irreducible representation of $\mathcal{G}(M)$, with $\boldsymbol{G}_m$ acting naturally (cf. Appendix to [6]).

Corollary. *Under the same notation as in above, let V and W be two subspaces of $H^i(M)$ with $V\supset W\neq\{0\}$. Assume that for any local ring $(B,\mathfrak{M})$ over k with the residue field k and any B-valued point λ of $\mathcal{G}(M)$, $U_\lambda(W\otimes B)\subset V\otimes B$. Then we have $V=H^i(M)$.*

Proof. Let $R=\Gamma(\mathcal{G}(M),\mathcal{O}_{\mathcal{G}(M)})$, and $\sigma: H^i(M)\to H^i(M)\otimes R$ be the co-module structure corresponding to the action U. We denote by γ the composition:

$$H^i(M)\otimes R^*\xrightarrow{\sigma\otimes 1_{R^*}} H^i(M)\otimes R\otimes R^*\xrightarrow{1_{H^i(M)}\otimes\langle\rangle} H^i(M),$$

where $R^*=\mathrm{Hom}_k(R,k)$ and $\langle\ \rangle$ stands for "contraction". Here we put $\overline{W}=\gamma(W\otimes R^*)$. Then obviously $\{o\}\neq W\subset\overline{W}\subset V$ and $\overline{W}$ becomes a stable subspace under the action U. Hence, by Mumford's theta structure theorem, we have $\overline{W}=V=H^i(M)$. Q.E.D.