47. *The Sheaf of Relative Canonical Forms of a Kähler Fiber Space over a Curve*

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In this note we announce an improvement of a result in [1]. Details shall be published elsewhere.

A triple $f: M\to S$ of a holomorphic mapping f and compact complex manifolds M, S is called a *Kähler fiber space* if M is Kähler, f is surjective and any general fiber of f is connected. By $\omega_{M/S}$ we denote the relative dualizing sheaf $\omega_M\otimes f^*\omega_S^\vee=\mathcal{O}_M[K_M-f^*K_S]$. Then we have the following

Theorem. *Let $f: M\to C$ be a Kähler fiber space over a curve C. Then $f_*\omega_{M/C}\cong\mathcal{O}_C[A\oplus U]$ for an ample vector bundle A and a flat vector bundle U on C.*

For a proof, we show the following lemma and use the criterion of Hartshorne [4].

Lemma. *Let E be the vector bundle such that $f_*\omega_{M/C}\cong\mathcal{O}_C[E]$. Then* $\deg(\det Q)\geqq 0$ *for any quotient bundle Q of E. Moreover, if* $\deg(\det Q)=0$, *then Q is a direct sum component of E and has a flat connection.*

Outline of the proof of lemma. Let S be the image of singular fibers of f and let $C^o=C-S$. Note that the restriction E_{C^o} of E to C^o is isomorphic to the bundle $\bigcup_{x\in C^o} H^{n,0}(F_x)$, where $F_x=f^{-1}(x)$ and $n=\dim F_x$. Hence E_{C^o} has a natural Hermitian structure. This defines a Hermitian structure of Q_{C^o} in a canonical manner. Let Ω be the Chern De Rham curvature form representing $c_1(Q_{C^o})$. Then we have the following formula: $\deg(\det Q)=\int_{C^o}\Omega+\sum_{p\in S}e_p$, where e_p is the local exponent of $\det Q$ at $p\in S$ (see [3]). Similarly as in [1], we prove that Ω is semi-positive and that $e_p\geqq 0$ for any $p\in S$. So $\deg(\det Q)\geqq 0$. Moreover, if $\deg(\det Q)=0$, then $\Omega\equiv 0$ and $e_p=0$ for any p. $\Omega\equiv 0$ implies that the orthogonal complements $\tilde{Q}_x$ $(x\in C^o)$ of $\mathrm{Ker}(E_x\to Q_x)$, considered as subspaces of $H^{n,0}(F_x)\subset H^n(F_x;\boldsymbol{C})$, form a flat subbundle of the flat bundle $\bigcup_{x\in C^o} H^n(F_x;\boldsymbol{C})$. So, Q_{C^o} is isomorphic to the vector bundle $\tilde{Q}_o=\bigcup_{x\in C^o}\tilde{Q}_x$ associated with the monodromy action of $\pi_1(C^o, x_o)$ on $\tilde{Q}_{x_o}\subset H^n(F_{x_o};\boldsymbol{C})$, where x_o is a point on C^o. Now, $e_p=0$